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APPLIED MECHANICS.

[Part 2.]

BY

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PRINTED BY JOHN LOVELL & SON.
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APPLIED MECHANICS.

CHAPTER I.

Frames.

(1). *Definitions.*—*Frames* are rigid structures composed of straight struts and ties, jointed together by means of bolts, straps, mortises and tenons, &c. *Struts* are members in compression, *ties* members in tension, and the term *brace* is applied to either.

The external forces upon a frame are the loads and the reactions at the points of support, from which may be found the resultant forces at the joints. The lines of action of these resultants intersect the joints at or near the centre in points called the *centres of resistance.* The figure formed by joining the centres of resistance *in order* is usually a polygon, and is designated the *line of resistance* of the frame.

The *position* of the centres should on no account be allowed to vary. It is assumed, and is practically true, that the joints of a frame are flexible, and that the frame under a given load does not sensibly change in form. Thus, an individual member is merely stretched or compressed in the direction of its length, *i.e.*, along its line of resistance, while the frame as a whole may be subjected to a bending action. The term *truss* is often applied to a frame supporting a weight.

(2) *Frame of two members.*

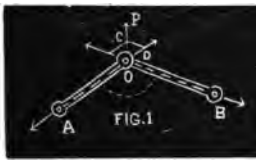


FIG. 1

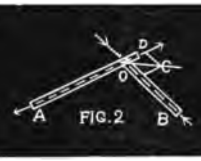


FIG. 2

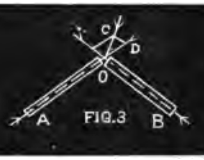


FIG. 3

OA, *OB* are two bars jointed at *O* and supported at the ends *A*, *B*. The frame in Fig. 1 consists of two ties, in Fig. 3 of two struts, and in Fig. 2 of a strut and a tie.

Let *P* be the resultant force at the joint, and let it act in the direction *OC*. Take *OC* equal to *P* in magnitude, and draw *CD* parallel to *OB*; *OD* is the stress along *OA*, and *CD* is that along *OB*.

Let the angle $AOB = a$, and the angle $COD = \beta$.

Let S_1, S_2, be the stresses along OA, OB, respectively.

$$\therefore\ \frac{S_1}{P} = \frac{OD}{OC} = \frac{\sin \overline{a-\beta}}{\sin a} \text{ and } \frac{S_2}{P} = \frac{CD}{OC} = \frac{\sin \beta}{\sin a}$$

(3.) *Frame of three or more members.*

FIG. 4

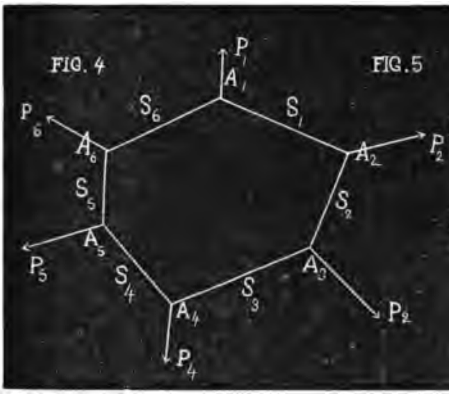
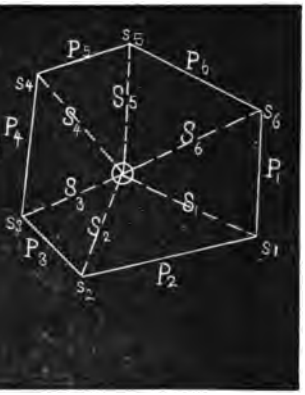


FIG. 5



Let $A_1A_2A_3$ … be a polygonal frame jointed at A_1, A_2, A_3…

Let P_1, P_2, P_3 … be the resultant forces at the joints A_1, A_2, A_3 … respectively. Let S_1, S_2, S_3 . …. be the forces along A_1A_2, $A_2\ A_3$ … respectively.

Consider the joint A_1.

The lines of action of three forces, P_1, S_1 and S_6, intersect in this joint, and the forces, being in equilibrium, may be represented in direction and magnitude by the sides of the triangle Os_1s_6, in which $s_1\ s_6$ is parallel to P_1, Os_1 to S_1, and Os_6 to S_6.

Similarly, P_2, S_1, S_2, may be represented by the sides of the triangle Os_1s_2 which has one side Os_1 common to the triangle Os_1s_6, and so on.

Thus, every joint furnishes a triangle having a side common to each of the two adjacent triangles, and all the triangles together form a *closed* polygon $s_1\ s_2\ s_3$… The sides of this polygon represent in magnitude and direction the resultant forces at the joints, and the radii from the pole O to the angles $s_1s_2s_3$ … represent in magnitude, direction and character, the forces along the several sides of the frame $A_1A_2A_3$ … The polygon $s_1s_2s_3$ … is the line of resistance of the frame, and is called the *funicular polygon* of the forces P_1, P_2, P_3, … with respect to the pole O.

Corollary 1.—The converse of the preceding is evidently true. For if a system of forces is in equilibrium, the polygon of forces $s_1\, s_2\, s_3 \ldots$ must close, and therefore the polygon which has its sides respectively parallel to the radii from a pole O to the angles s_1, s_2, s_3, ... and which has its angles upon the lines of action of the forces, must also close.

Corollary 2.—Let the resultant forces at the joints be parallel.

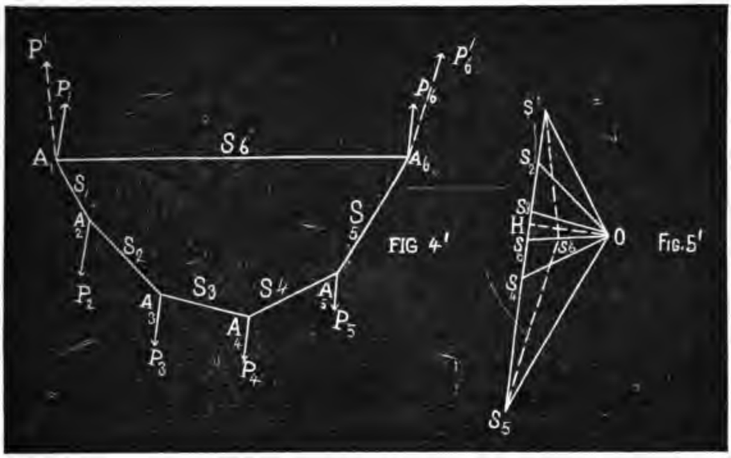


FIG 4′

FIG. 5′

The polygon of forces becomes the straight line $s_1 s_5$, which is often termed the *line of loads*. Thus, the forces P_2, P_3, $\ldots P_5$ are represented by the sides $s_1\, s_2$, $s_2\, s_3$, ... $s_4\, s_5$, which are in one straight line *closed* by $s_1\, s_6$ and $s_5\, s_6$, representing the remaining forces P_1 and P_6.

Draw OH perpendicular to $s_1 s_5$, the line of loads; OH represents in magnitude and direction the stress which is the same for each member of the frame.

Let a_1, a_2, $a_3 \ldots$ be the inclinations of the members $A_1\, A_2$, $A_2\, A_3, \ldots$, respectively, to the line of loads.

$$\therefore\ OH = Hs_1 . \tan a_1 \text{ and also, } OH = Hs_5 . \tan a_5$$

$$\therefore\ OH. (\cot a_1 + \cot a_5) = Hs_1 + Hs_5 = s_1\, s_5 = P_2 + P_3 + P_4 + P_5 = P_1 + P_6$$

$$\therefore OH = \frac{P_2 + \ldots + P_5}{\cot a_1 + \cot a_5} = \frac{P_1 + P_6}{\cot a_1 + \cot a_5}$$ and is the stress common to each member.

Again, the stress in any member, *e.g.*, $A_4\, A_5$, is $Os_4 = \dfrac{OH.}{\sin a_4}$

Corollary 3.—Let the resultant forces at the joints A_1, A_6, be inclined to the common direction of the remaining forces, and act in the directions shewn by the dotted lines (Fig. 4′). Let P'_1, P'_6 be the magnitudes of the new forces; draw $s_1 s'_6$ parallel to the direction of P'_1 so as to meet Os_6 *in* s'_6, (Fig. 5); join $s_5 s'_6$. Since there is equilibrium, $s'_6 s_5$ must be parallel to the line of action of P'_6.

Thus $s_1 s'_6 s_5$ is the force polygon.

Corollary 4.—The forces, or loads, P_2, $P_3 \ldots P_5$, are generally vertical, while P_1, P_6, are the vertical reactions of two supports. In this case, s_6 and H coincide, and $O\ H$ is the horizontal stress common to each member of the frame.

The stress in any bar may be found as in Corollary 2.

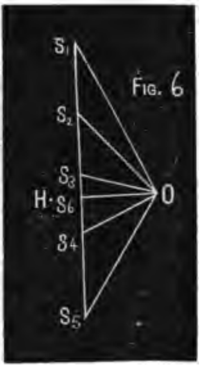


FIG. 6

Corollary 5.—If the number of the forces P_1, P_2, ... is increased indefinitely, and the distance between the lines of action of the forces indefinitely diminished, the funicular polygon becomes a curve and is called the *funicular curve*.

Corollary 6.—If more than *two* members meet at a joint, or if the joint is subjected to more than *one* load, the resulting force diagram will be a quadrilateral, pentagon, hexagon ... according as the number of members is 3, 4, 5 ... or the number of loads is 2, 3, 4...

(4) *Method of Sections.* It often happens that the stresses in the members of a frame may be easily obtained by the method of sections. This method is precisely similar to that employed in discussing the equilibrium of beams, and depends upon the following principle:—

If a frame is divided by a plane section into two parts, and if each part is considered separately, the stresses in the bars (or members) intersected by the secant plane must balance the external forces upon the part in question.

Hence, the *algebraic* sums of the horizontal components, of the vertical components, and of the moments of the forces with respect to any point, are severally zero, *i.e.*, analytically,

$$\Sigma(X)=o,\ \Sigma(Y)=o, \text{ and } M=O$$

These equations are solvable if they do not involve more than three unknown quantities, *i.e.*, if the secant plane does not cut more than three members, the corresponding stresses are determinate.

(5) *Jib-Crane.* Fig. 7 is a skeleton diagram of an ordinary jib-crane.

OA is the post fixed in the ground at *O*.

OB is the jib, AB is the tie.

The jib, tie and gearing are suspended from the top of the post by a cross-head, which admits of a free rotation round the axis of the post.

Let the crane raise a weight W.

Three forces in equilibrium meet at B, viz., W, the tension T in the tie, and the thrust C along the jib. They may be represented by the sides of the triangle AOB to which they are respectively parallel.

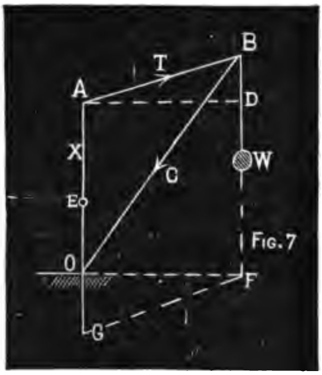


FIG. 7

$$\therefore T = W.\frac{AB}{AO} \text{ and } C = W.\frac{BO}{AO}$$

The stress S in the chain tends to produce a compressive stress of equal magnitude in any member along which it passes. Thus, if it pass along AB the resultant stress in the tie is $T - S$, while if it pass along BO, the resultant stress in the jib is $C + S$.

Also, *neglecting friction*, $S = \frac{W}{n}$, n being the number of falls of chain from B.

Again, the post is a semi-girder fixed at one end and acted upon at the other by a force T.

Let M_x be the bending moment at any point E of the post distant x from A.

$$\therefore M_x = T.\ Cos\, BAD.\ x = W.\frac{AB}{AO}.\frac{AD}{AB}.\ x = W.\frac{AD}{AO}.\ x$$

The bending moment at O is evidently $W.AD$, and is independent of the height of the post.

The vertical component of T, viz., $T.\frac{BD}{AB} = W.\frac{BD}{AO}$, is transmitted through the post. Hence, the total unit-stress in the layers of the transverse section at E, distant y from the axis, is $\frac{W}{A}.\frac{BD}{AO} \pm \frac{y}{I} M_x$, A being the area of the section, and I its moment of inertia with respect to the axis. The total resultant pressure along the post at O

$$= -T.\sin BAD + C.\sin BOF = -W.\frac{BD}{AO} + W.\frac{BO}{AO}.\frac{BD + AO}{BO} = W$$

If the post revolves about its axis, the jib and gearing are bolted to it, and the whole turns on a pivot at the toe G. In this case the frame, as a whole, is kept in equilibrium by the weight W, the horizontal reaction H of the curb-plate at O, and the reaction R at G. The first two forces meet in F, and therefore the reaction at G must also pass through F.

Hence, since OFG is a triangle of forces,

$$H = W.\frac{OF}{OG}, \text{ and } R = W.\frac{GF}{OG}.$$

(6) *Derrick-Crane.* Fig. 8, shews a combination of a derrick and crane called a derrick-crane. It is distinguished from the jib-crane, by having two *back-stays*, which are usually situated in planes at right-angles to each other. One end of the jib is hinged at or near the foot of the post, and the other is held by a chain, which passes over pulleys to a winch on the post, so that the jib may be raised or lowered, as required.

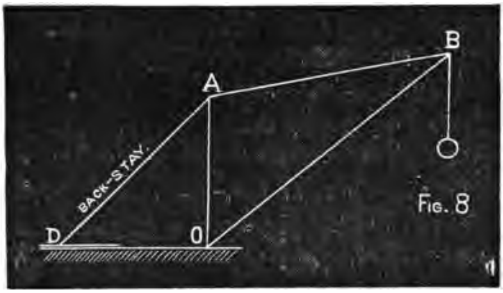


Fig. 8

The derrick-crane is generally of wood, is simple in construction, is easily erected, has a vertical as well as a lateral motion, and a range equal to a circle of from 10 to 60 ft. in radius. It is, therefore, very useful for temporary works, setting masonry, &c.

The stresses in the jib and tie are obtained as in §(5), and those in the back-stays and post may be calculated as follows:—

Let OH, OK, OL, be the horizontal traces of the tie and back-stays.

In HO produced, take ON to represent the horizontal component of the stress in the tie, *i.e.*, $T.\sin a$, a being the inclination of the tie to the vertical. Complete the parallelogram XY.

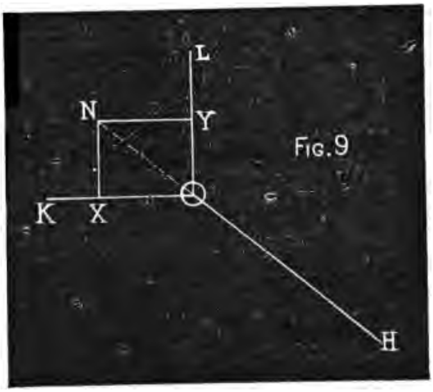


Fig. 9

OX and OY are the horizontal components of the stresses in the back-stays. Let the angle $NOX = \theta$.

$\therefore OX = ON.\cos\theta = T.\sin a.\cos\theta$, and is a maximum when $\theta = 0^\circ$

$OY = ON.\sin\theta = T.\sin a.\sin\theta$, " " " $\theta = 90^\circ$

Hence, the stress in the back-stay is greatest, when the plane of the back-stay and post coincides with that of the jib and tie.

Again, let β be the inclination of the back-stays to the vertical. The vertical components of the back-stay stresses are $T.\sin a.\cos\theta.\cot\beta$ and $T.\sin a.\sin\theta.\cot\beta$, so that the corresponding stress along the post is $T.\sin a.\cot\beta.(\cos\theta + \sin\theta)$, which is a maximum when $\theta = 45^\circ$.

(7). *Bent Crane.*

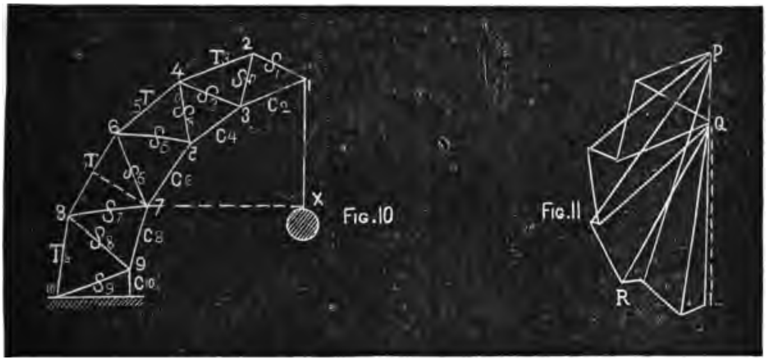


Fig. 10. shews a convenient form of crane when much head-room is required near the post. The crane is merely a semi-girder, and may be tubular with plate webs if the loads are heavy, or its flanges may be braced together as in the Fig. for loads of less than 10 tons. The flanges may be kept at the same distance apart throughout, or the distance may be gradually diminished from the base towards the peak.

Let the letters in Fig. 10 denote the stresses in the corresponding members. *Three* forces S_1, C_2, and W, act through the point (1), so that S_1 and C_2 may be obtained in terms of W; *three* forces S_1, S_2, T_3, act through (2), so that S_2 and T_3 may be obtained in terms of S_1 and therefore of W; *four* forces S_2, C_2, S_3, C_4, act through (3), and the values of S_2, C_2, being known, those of S_3, C_4, may be determined. Proceeding in this way, it is found that of the forces at each succeeding joint, only two are unknown, and the values of these are consequently determinate.

The calculations may be checked by the *method* of *moments* and by the stress diagram, (Fig. 11).

e.g., let $W = 10$ tons.

Take moments about the point (7).

$$\therefore T_7 . (y7) = 10 . (x7) \quad \text{or} \quad T_7 = \frac{3.25}{1.25} . 10 = 26 \text{ tons.}$$

No other forces enter into the equation of moments, as the portion of the crane above a plane intersecting 68 and passing through (7) is kept in equilibrium by the weight of 10 tons and the stresses T_7, S_6, C_6; the moments of S_6 and C_6 about (7) are evidently zero.

Again, from the stress diagram, $T_7 = QR = \frac{3.9}{1.5} . 10 = 26$ tons.

(8) *Timber bridge and roof trusses of small span.*

(*a*).—A single girder is the simplest kind of bridge, but is only suitable for very short spans. When the spans are wider, the centre of the girder may be supported by struts *OC*, *OD*, Fig. 12, through which a portion of the weight is transmitted to the abutments.

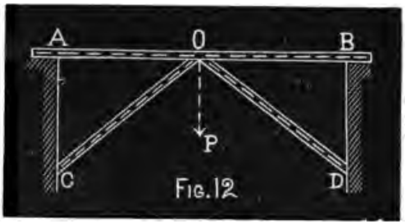


Fig. 12

Let the weight at *O* be *P*, and let *a* be the angle *AOC*.

The thrust along $OC = \frac{P}{2} . \frac{OC}{AC} = \frac{P}{2} . \frac{1}{\sin a}$

The horizontal thrust at $O = \frac{P}{2} . \frac{OA}{AC} = \frac{P}{2} . \cot a$

The horizontal and vertical thrusts upon the masonry at *C* (or *D*), are $\frac{P}{2} . \cot a$, and $\frac{P}{2} . \frac{1}{\sin a}$, respectively.

If the girder is uniformly loaded, *P* is *one-half* of the whole load.

(*b*).—In Fig. 13 a *straining cill*, *E F*, is introduced, and the girder is supported at two points.

Let *P* be the weight at each of the points *E* and *F*.

The thrust in *EC* (or *FD*) $= P . \frac{EC}{AC}$, and the horizontal thrust in the straining piece $= P . \frac{AE}{AC}$.

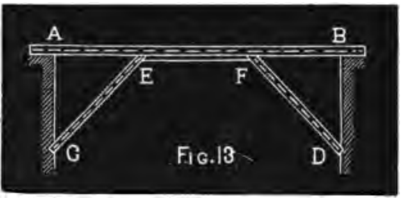
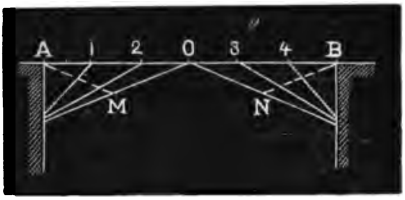


Fig. 13

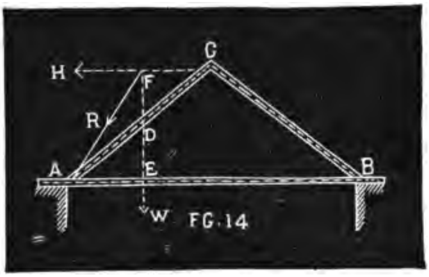
If a load is uniformly distributed over *AB*, it may be assumed that each strut carries one-half of the load upon *AF* (or *BE*), and that each abutment carries one-half of the load upon *AE* (or *BF*).



By means of straining cills the girders may be supported at several points 1, 2, ... and the weight concentrated at each may be assumed to be one-half of the load between the two adjacent points of support. The calculations for the stresses in the struts, etc... are made precisely as above.

If the struts are very long they are liable to bend, and counterbraces *AM*, *BN*, are added to counteract this tendency.

(*c*).—The triangle is the only geometrical figure of which the form cannot be changed without varying the lengths of the sides. For this reason, all compound trusses for bridges, roofs, etc., are made up of triangular frames.



FG. 14

Fig. 14 represents the simplest form of roof-truss, the dotted lines being the lines of resistance of the bars.

AC, *BC* are rafters of equal length inclined to the horizontal at an angle *a*, and each carries a uniformly distributed load *W*.

The rafters react horizontally upon each other at *C*, and their feet are kept in position by the tie-beam *AB*.

Consider the rafter *AC*.

The resultant of the load upon *AC*, *i.e.*, *W*, acts through the middle point *D*.

Let it meet the horizontal thrust *H* of *BC* upon *AC* in *F*.

For equilibrium, the resultant thrust at *A* must also act through *F*.

The sides of the triangle *AFE*, evidently represent the three forces.

Hence, $$H = W.\frac{AE}{EF} = \frac{W}{2}.\frac{AE}{DE} = \frac{W}{2}.\cot a.$$

$$R = W.\frac{AF}{FE} = W.\sqrt{\frac{AE^2 + EF^2}{FE^2}} = W.\sqrt{1 + \frac{1}{4}.\frac{AE^2}{DE^2}}$$
$$= W.\sqrt{1 + \frac{1}{4}.\cot^2 a.}$$

The thrust *R* produces a tension *H* in the tie-beam, and a vertical pressure *W* upon the support.

Also, if γ is the angle FAE,

$$\tan\gamma = \frac{EF}{AE} = 2.\ \frac{DE}{AE} = 2.\ \tan a.$$

If the rafters AC, BC, are unequal, let a_1, a_2, be their inclination to AB, respectively.

Let W_1 be the uniformly distributed load upon AC, W_2 that upon BC.

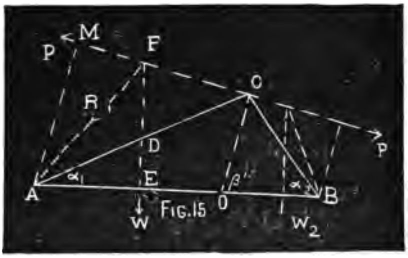


Fig. 15

Let the direction of the mutual thrust P at C make an angle β with the vertical so that if CO is drawn perpendicular to FC, the angle $COB = \beta$; the angle $ACF = 90° — ACO = 90° — \overline{\beta - a_1}$

Draw AM perpendicular to the direction of P, and consider the rafter AC. As before the thrust R_1 at A, the resultant weight W_1 at the middle point D of AC, and the thrust P at C, meet in the point F. Take moments about A,

$$\therefore P.\ AM = W_1.\ AE$$

But $AM = AC.\ \sin ACM = AC.\ \cos \overline{\beta - a_1}$, and $AE = \frac{AC}{2}.\ \cos.\ a_1$,

$$\therefore P = \frac{W_1}{2}.\ \frac{\cos a_1}{\cos\ (\beta - a_1)}$$

Similarly, by considering the rafter BC,

$$P = \frac{W_2}{2}.\ \frac{\cos d_2}{\sin\ (\beta + a_2 - 90)} = -\frac{W_2}{2}.\ \frac{\cos a_2}{\cos\ (\beta + a_2)}$$

Hence, $\frac{W_1}{2}.\ \frac{\cos a_1}{\cos\ (\beta - a_1)} = P = -\frac{W_2}{2}.\ \frac{\cos a}{\cos\ (\beta + a_2)}$

and $\therefore\ \tan\beta = \frac{W_1 + W_2}{W_1.\ \tan a_2 - W_2.\ \tan a_1.}$

The horizontal thrust of each rafter $= P.\ \sin\beta$

The vertical thrust upon the support $A = W_1 - P.\ \cos\beta$

" " " " $B = W_2 + P.\ \cos\beta$

(*d*).—*King-post truss.* The simple triangular truss may be modified by introducing a king-post CO, (Fig. 16), which carries a portion of the weight of the beam AB, and transfers it through the rafters so as to act upon the tie in the form of a tensile stress.

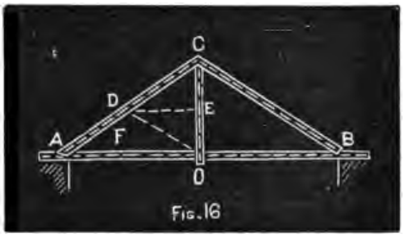


Fig. 16

Let P be the weight borne by the king-post; represent it by CO.

Draw OD parallel to BC and DE parallel to AB.

$DC = \dfrac{CE}{\sin a} = \dfrac{P}{2} \cdot \dfrac{1}{\sin a}$, is the thrust in CA due to P, and is of course equal to DO, *i. e.*, the thrust along CB.

$DE = CE . \cot a = \dfrac{P}{2} . \cot a$, is the horizontal thrust on each rafter, and is also the tension in the tie due to P.

Let W be the uniformly distributed load upon each rafter.

The total horizontal thrust upon each rafter $= (W + P) . \dfrac{\cot a}{2}$

The total vertical pressure upon each support $= W + \dfrac{P}{2}$

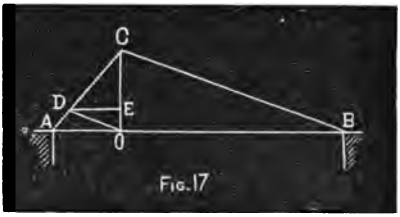


FIG. 17

If the apex C is not vertically over the centre of the tie-beam, (Fig. 17), take CO, as before, to represent the weight P borne by the king-post; draw OD parallel to BC, and DE parallel to AB.

The weight P produces a thrust CD along CA, DO along CB, and a horizontal thrust DE upon each rafter

CE is the portion of P supported at A and EO that supported at B.

DE, and therefore the tension in the tie AB, diminishes with AO, being zero when AC is vertical.

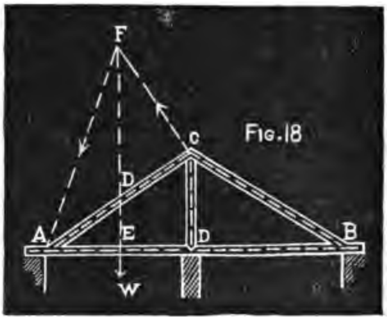


FIG. 18

Sometimes it is expedient to support the centre of the tie-beam upon a column or wall (Fig. 18), the king-post being a pillar against which the heads of the rafters rest.

Consider the rafter AC.

The normal reaction R' of CO upon AC, the resultant weight W at the middle point D, and the thrust R at A meet in the point F.

Take moments about A,

$$\therefore R' . AC = W . AE, \text{ or } R' = \frac{W}{2} . \cos a$$

Thus the total thrust transmitted through CO to the support at O is

$2 . \dfrac{W}{2} \cos a . \cos a = W . \cos_2 a$

The horizontal thrust upon each rafter $= \frac{W}{2} . \cos a . \sin a = \frac{W . \sin 2 a.}{4}$

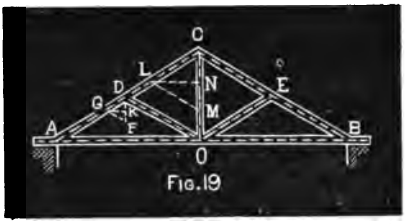


FIG. 19

(*e*).—If the rafters are inconveniently long, or if they are in danger of bending and breaking across, the centres may be supported by struts *OD*, *OE*, (Fig. 19). A portion of the weight upon the rafters is transmitted through the struts to the king-post (or rod), which again transmits it through the rafters to act partly as a vertical pressure upon the supports, and partly as a tension on the tie-beam.

Indeed, the main duty of struts and ties is to transform transverse into longitudinal stresses.

The load upon the rafters ($2.W$) may be assumed to be concentrated at the points *A*, *D*, *C*, *E*, *and B*, in the proportions $\frac{W}{4}$, $\frac{W}{2}$, $\frac{W}{2}$, $\frac{W}{2}$, and $\frac{W}{4}$, respectively. Drop the vertical *DF* to represent the load $\frac{W}{2}$ at *D*; draw *FG* parallel to *OD*, and *GK* parallel to *AB*.

GF ($= DG$) is the thrust along *DO*, and may be resolved at *O* into its vertical ($= DK$) and horizontal ($= GK$) components; the latter is counteracted by an equal horizontal thrust from the strut *OE*. The vertical component $DK\left(=\frac{W}{4}\right)$ is borne by the king-post, so that the total pull upon the king-post is $\frac{W}{2} + P$, P including the weight of the struts and post.

Take *CM* to represent this pull *plus* the weight at *C*, *i.e.*, $CM = W + P$. Draw *ML* parallel to *BC*, and *LN* parallel to *AB*.

$CL = \frac{W + P}{2} . \frac{1}{\sin a}$ is the thrust along the rafter from *C* to *D*.

$CL + DG = \left(\frac{3.W}{4} + \frac{P}{2}\right) . \frac{1}{\sin a}$, is the thrust along the rafter from *D* to *A*.

$LN + GK = \left(\frac{3.W}{4} + \frac{P}{2}\right) . \cot a$, is the tension in the tie.

NOTE.—The king-post truss is the simplest and most economical frame for spans of less than 30 ft. In larger spans, two or more suspenders may be introduced, or the truss otherwise modified.

Collar-beams (*DE*), queen-posts (*DF*, *EG*), braces, &c..., (Figs. 20, 21), may be employed to prevent the deflection of the rafters, and the complexity of the truss necessarily increases with the span and with the weight to be borne.

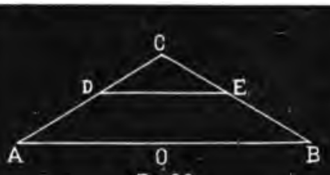


FIG. 20

The stresses in the several members are calculated in the manner already described.

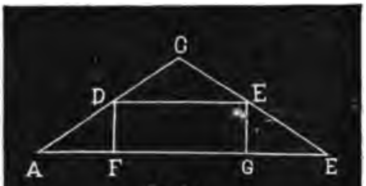


FIG. 21

In Fig. 21, e. g., it may be assumed that one-half of the weight upon $AC = \left(\frac{W}{2}\right)$, is concentrated at *D*, and that the component of this weight perpendicular to the rafter viz., $\frac{W}{2} \cdot \cos a$, is distributed between the collar-beam *DE* and the queen-post *DF*.

(9) *Trusses for bridges and roofs of large span.*

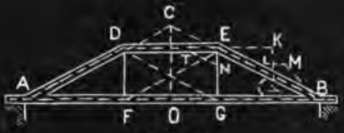


FIG. 22

In bridges and roofs of large span the rafters or braces, *AC*, *BC*, are very long, and the unsupported distance between *A* and *O*, or *B* and *O*, is too great. The rafters are therefore cut off at *D* and *E*, (Fig. 22), a straining piece *DE* is introduced, and the tie-beam *AB* is supported at two points by queen-rods (or posts), *DF* and *EG*.

Let *W* be the load upon each of the braces *AD*, *BE*.

Let *P* be the weight borne by each queen-rod.

The thrust along *DE*, the resultant weight upon *EB*, and the resultant thrust at *B* evidently meet in the point *K*.

Take *KL* to represent *W*, and draw *LM* parallel to *EK*.

LM is the horizontal thrust in *DE*, or the tension in *AB*, due to *W*.

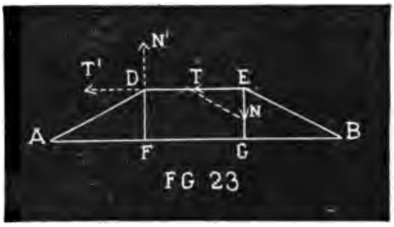
Take *EN* to represent *P*, and draw *NT* parallel to *EB*.

ET is the thrust in *ED*, or the tension in *AB*, due to *P*.

Hence, the total thrust in DE, or tension in AB, $= LM + ET$ $= \left(\frac{W}{3} + P\right). \cot a.$

The values of P and W depend upon the character of the loading.

If the load, including the weight of the truss, is uniformly distributed over the tie-beam, as in the case of a bridge, it may be assumed to be concentrated at the points A, F, G and B, in the proportions *one-half* of the load upon AF, one-half of the load upon AG, one-half of the load upon BF, and one-half of the load upon BG, respectively.



FG 23

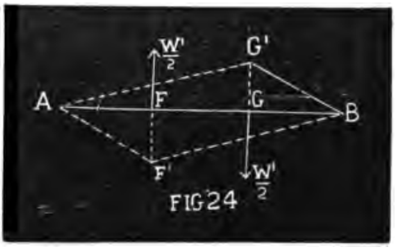


FIG 24

Suppose the load to be unequally distributed, and that a single weight W' is suspended from E. This produces an upward pull at D, equal in magnitude to the downward pull at E.

The latter force may be neutralized by an equal and opposite force at E, or by a downward pull of $\frac{W'}{2}$ at D, together with an upward pull of $\frac{W'}{2}$ at E, which is the manner in which the beam AB neutralizes the effect of the weight W'.

The beam AB is therefore acted upon by two opposite forces, the one at F, the other at G, each being equal to $\frac{W'}{2}$ in magnitude.

Let R be the reaction at A; let $AB = l$, and $FG = c$.

$$\therefore R.l = -\frac{W'}{2}.\frac{l+c}{2} + \frac{W'}{2}.\frac{l-c}{2} = -\frac{W'}{2}.c, \text{ and } R = -\frac{W'}{2}.\frac{c}{l}$$

The magnitude of the bending moment at F and G, at which points it is evidently a maximum, is $\frac{W'}{2}.\frac{c}{l}.\frac{l-c}{2} = \frac{W'}{4}.c.\frac{l-c}{l}$.

The ordinates of the dotted lines, Fig. 24, are proportional to the bending moments due to the two equal and opposite forces—FF' being the bending moment due to the upward pull at D, and GG' that due to the downward pull at E.

Again, the rigidity of the truss may be increased by introducing diagonal braces DG and EF, as shewn by the dotted lines in Fig. 22.

The shearing force for points between F and G is a maximum when the load is at F or G, its value being $\pm \frac{W'}{2l} . c \mp \frac{W'}{2} = \mp \frac{W'}{2} . \frac{l-c}{l}$. This force has to be transmitted to the horizontal beams through the medium of a diagonal, which is DG when the load is at F and EF when the load is at G. Thus, the magnitude of the greatest thrust in either of the diagonals is, $\frac{W'}{2} . \frac{l-c}{l} . \frac{s}{d}$,

s being the length of a diagonal, and d the depth of the truss.

(10) *General remarks.*—In the trusses described in (*d*) and (*e*) § (8), and in § (9), the vertical members are ties, *i.e.*, are in tension, and the inclined members are struts, *i.e.*, are in compression. By inverting the respective figures another type of truss is obtained in which the verticals are struts while the inclined members are ties. Both systems are widely used, and the method of calculating the stresses is precisely the same in each.

In designing any particular member, allowance must be made for every kind of stress to which it may be subjected. The collar-beam DE in Fig. 21, for example, must be treated as a pillar subjected to a thrust in the direction of its length at each end; if it carry a transverse load, its strength as a beam, supported at the points D and E, must also be determined. Similarly, the rafters AC, BC, Fig. 19, etc., must be designed to carry transverse loads and to act as pillars. But it must be remembered that struts and queen-posts provide additional points of support over which the rafters are continuous, and it is therefore practically sufficient to assume that the rafters are divided into a number of short lengths, each of which carries *one-half* of the load between the two adjacent supports.

When a tie-beam is so long as to require to be spliced, allowance must be made for the weakening effect of the splice.

(11) *Compound Roofs.*—In mansard roofs and many other framings a number of rafters are made to abut against each other as in Fig 25.

The relative position of the rafters is fixed by the condition that the horizontal pressure upon each must be the same,

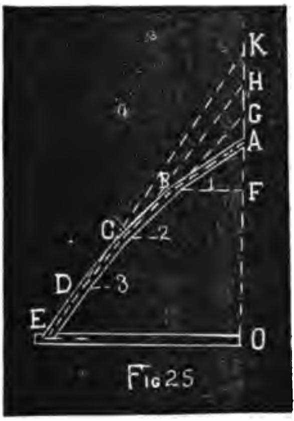


FIG 25

in order that the pressure may be completely transmitted from one rafter to the next.

Let W_1, W_2, W_3 ..., a_1, a_2, a_3 ... be the weights and inclinations to the horizontal of the rafters AB, B,C, CD... respectively.

Assume that the weights are concentrated at the points A, B, C...... in the proportions $\frac{W_1}{2}$, $\frac{W_1 + W_2}{2}$, $\frac{W_2 + W_3}{2}$,......, respectively.

The triangles BFA, BAG, CGH,... represent the forces at these points.

Hence the horizontal pressure upon $AB = \frac{W_1}{2} \cdot \frac{BF}{AF} = \frac{W_1}{2} \cdot \cot a_1 = H_1$

The resultant thrust along $BC = \frac{W_1 + W_2}{2} \cdot \frac{BG}{GA} = \frac{W_1 + W_2}{2} \cdot \frac{\cos a_1}{\sin(a_2 - a_1)}$

and its horizontal component $= \frac{W_1 + W_2}{2} \cdot \frac{\cos a_1 \cos a_2}{\sin(a_2 - a_1)} = H_2$.

Similarly the horizontal component of the thrust along CD

$$= \frac{W_2 + W_3}{2} \cdot \frac{\cos a_2 \cos a_3}{\sin(a_3 - a_2)} = H_3,$$ and so on.

But $H_1 = H_2 = H_3$ . . . .

$$\therefore \frac{W_1}{2} \cdot \cot a_1 = \frac{W_1 + W_2}{2} \cdot \frac{\cos a_1 . \cos a_2}{\sin(a_2 - a_1)} = \frac{W_2 + W_3}{2} \cdot \frac{\cos a_2 \cos a_3}{\cos(a_3 - a_2)} \quad . . .$$

or $$\frac{W_1}{2} \cdot \cot a_1 = \frac{W_1 + W_2}{2} \cdot \frac{2}{\tan a_2 - \tan a_1} = \frac{W_2 + W_3}{2} \cdot \frac{1}{\tan a_3 - \tan a_2} = - - -$$

Hence, $$\tan a_2 = \left(2 + \frac{W_2}{W_1}\right) . \tan a_1$$

$$\tan a_3 = \left(2 + \frac{2 W_2}{W_1} + \frac{W_3}{W_1}\right) . \tan a_1$$

$$\tan a_4 = \left(2 + \frac{2 W_2 + 2 W_3 + W_4}{W_1}\right) . \tan a_1$$

&c. &c.

If $W_1 = W_2 = W_3 \ldots, \therefore \tan a_1 = \frac{1}{3} \cdot \tan a_2 = \frac{1}{5} \cdot \tan a_3 = \frac{1}{7} \cdot \tan a_4 = \ldots\ldots$

(12) *Gothic Roof Truss.*—This class of truss exerts an oblique pressure upon the supporting walls, and the framing should be so designed as to reduce the horizontal component of this pressure to a minimum.

The primary truss $K\ D\ E\ L$ supports the weights at D, C, and E, (Fig. 26).

The weight W upon AC, or BC, may be assumed to be concentrated at the points A, D, C, E, and B, in the proportions $\frac{W}{4}$, $\frac{W}{2}$, $\frac{W}{2}$, $\frac{W}{2}$,

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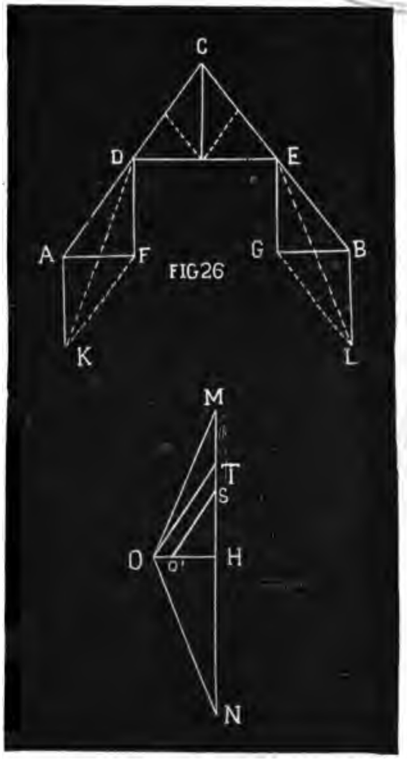


FIG 26

and $\frac{W}{4}$, respectively. Hence the total weight upon the primary truss $= \frac{3}{2}.W$. Trace the stress diagram; the *line of loads* MN is evidently vertical; take $MN = \frac{3}{2} \cdot W$; draw OM, ON, parallel to DK, EL, respectively.

The horizontal line OH is the thrust along DE, and OM is the thrust along DK.

Draw OT parallel to AD; the sides of the triangle OTH represent the stresses in the members AD, DF, and FA.

Also, since DF is equal and parallel to AK, T is the middle point of MH.

The thrust along AK = the thrust along DF + portion of weight concentrated at A

$$= \frac{W}{4} + TH = \frac{W}{4} + \frac{3}{8}. W = \frac{5}{8}. W.$$

The secondary truss DCE carries a weight $\frac{W}{2}$ at C

Take $HS = \frac{W}{4}$ and draw SO' parallel to CD; $SO'H$ is a triangle of forces for one-half of the secondary truss.

Also, $\frac{O'H}{OH} = \frac{HS}{HT} = \frac{\frac{W}{4}}{\frac{3}{8}. W} = \frac{2}{3}$, and the resultant thrust along DE is

$$OH - O'H = \frac{1}{3}. OH.$$

CHAPTER II.

Roofs.

(1).—*Types of Truss.*—A roof truss may be constructed of timber, of iron, or of timber and iron combined. Timber is almost invariably employed for small spans, but in the longer spans it has been largely superseded by iron, in consequence of the combined lightness, strength and durability of the latter.

Attempts have been made to classify roofs according to the mode of construction, but the variety of form is so great as to render it impracticable to make any further distinction than that which may be drawn between those in which the reactions of the supports are vertical and those in which they are inclined.

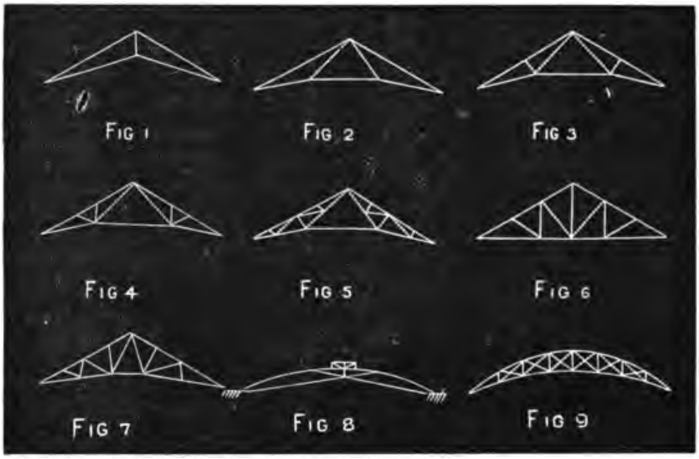


Fig. 1 is the simplest form of truss for spans of less than 30-ft.

Fig. 2 is a superior framing for spans of from 30 to 40–ft.; it may be still further strengthened by the introduction of struts, Figs. 3 and 4, and with such modification has been employed to span openings of 90 ft. It is safer, however, to limit the use of the type shewn by Fig. 3 to spans of less than 60–ft. Figs, 5, 6, 7, 8 and 9 are forms of truss suitable for spans of from 60 to 100–ft. and upwards.

Arched roofs (Figs. 8 and 9) admit of a great variety of treatment.

They have a pleasing appearance, and cover wide spans without intermediate supports. The flatness of the arch is limited by the requirement of a minimum thrust at the abutments. The thrust may be resisted either by thickening the abutments or by introducing a tie. If the only load upon a roof-truss were its own weight, an arch in the form of an inverted catenary, with a shallow rib, might be used. But the action of the wind induces oblique and transverse stresses, so that a considerable depth of rib is generally needed. If the depth exceed 12–ins., it is better to connect the two flanges by braces than by a solid web.

Roofs of wide span are occasionally carried by ordinary lattice girders.

(2) *Principals, Purlins, &c.*

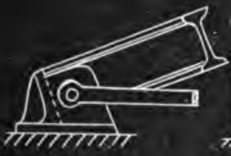


Fig 10

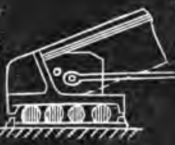


Fig 11

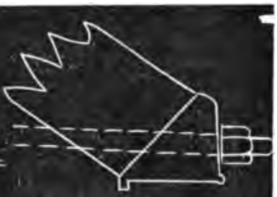


Fig 12

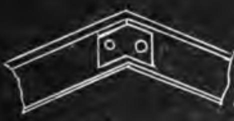


Fig 13

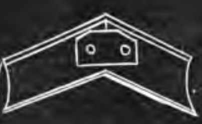


Fig 14

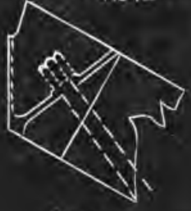


Fig 15

The principal rafters in Figs. 1 to 7 are straight, abut against each other at the peak, and are prevented by tie-rods from spreading at the heels. When made of iron, tee (T), rail, and channel (both single ⊔ and double][ ) bars, bulb tee (T) and rolled (I) iron beams, are all excellent forms.

Timber rafters are rectangular in section, and for the sake of economy and appearance, are often made to taper uniformly from heel to peak.

The heel is fitted into a suitable cast-iron skew-back, or is fixed between wrought-iron angle brackets, (Figs. 10, 11, 12), and rests either directly upon the wall or upon a wall-plate.

When the span exceeds 60–ft., allowance should be made for alterations of length due to changes of temperature. This may be effected by interposing a set of rollers between the skew-back and wall-plate at one heel, or by fixing one heel to the wall and allowing the opposite skew-back to slide freely over a wall-plate.

The junction at the peak is made by means of a casting or wrought iron plates, (Figs. 13, 14, 15).

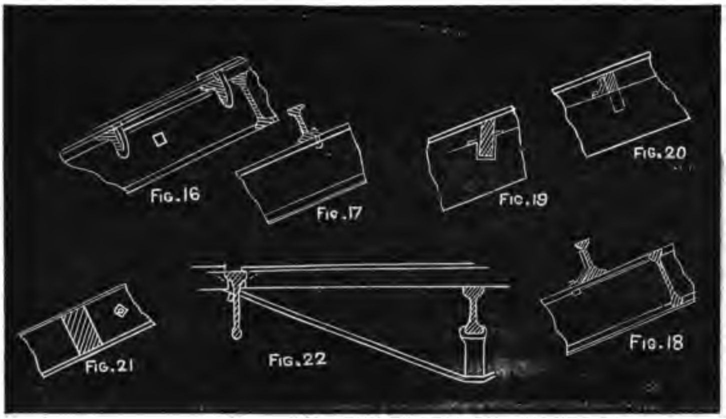
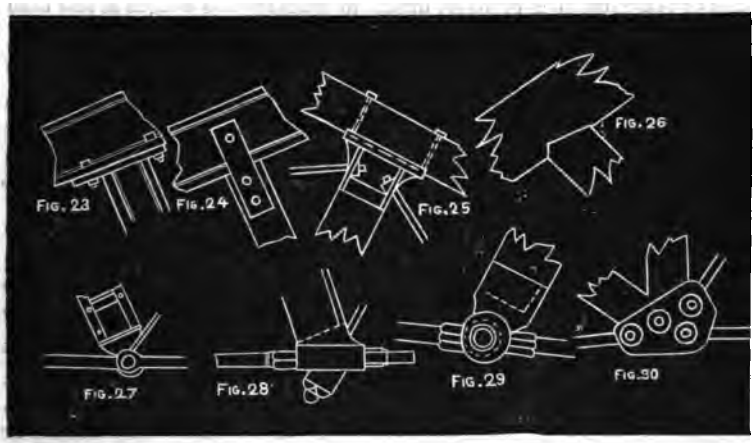


FIG. 16 FIG. 17 FIG. 19 FIG. 20 FIG. 21 FIG. 22 FIG. 18

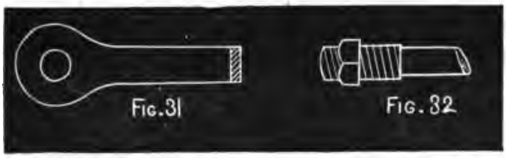
Light iron and timber beams as well as angle-irons are employed as purlins. They are fixed to the top or sides of the rafters by brackets, or lie between them in cast-iron shoes, (Figs. 16 to 21), and are usually held in place by rows of tie-rods, spaced at 6 or 8 ft. intervals between peak and heel, running the whole length of the roof.

The sheathing boards and final metal or slate covering is fastened upon the purlins. The nature of the covering regulates the spacing of the purlins, and the size of the purlins is governed by the distance between the main rafters, which may vary from 4-ft. to upwards of 25-ft. But when the interval between the rafters is so great as to cause an undue deflection of the purlins, the latter should be trussed. Each purlin may be trussed, or a light beam may be placed midway between the main rafters so as to form a supplementary rafter, and trussed as in Fig. 22.

Struts are made of timber or iron. Timber struts are rectangular in section. Wrought iron struts may consist of L irons, T-bars, or light columns, while cast-iron may be employed for work of a more ornamental character. The strut-heads are attached to the rafters by means of cast caps, wrought-iron straps, brackets, &c., (Figs. 23 to 26), and the strut-feet are easily designed both for pin and screw connections, (Figs. 27 to 30).



Ties may be of flat or round bars attached either by eyes and pins or by screw ends, and occasionally by rivets, (Figs. 31, 32). The greatest care is necessary in properly proportioning the dimensions of the eyes and pins to the stresses that come upon them.



To obtain greater security, each of the end panels of a roof may be provided with lateral braces, and wind-ties are often made to run the whole length of the structure through the feet of the main struts.

Due allowance must be made in all cases for changes of temperature.

(3) *Roof weights.*—In calculating the stresses in the different members of a roof truss two kinds of load have to be dealt with, the one *permanent* and the other *accidental.*

The permanent load consists of the *covering*, the *framing*, and *accumulations* of *snow.*

Tables at the end of the chapter shew the weights of various coverings and framings.

The weight of freshly fallen snow may vary from 5 to 20-lbs. per cubic ft. English and European engineers consider an allowance of 6 lbs. per sq. ft. sufficient for snow, but in cold climates, similar to that of North America, it is probably unsafe to estimate this weight at less than 12–lbs. per sq. ft. .

The *accidental* or *live* load upon a roof is the wind pressure, the maximum force of which has been estimated to vary from 40 to 50–lbs. per sq. ft. of surface *perpendicular to the direction of blow*. Ordinary gales blow with a force of from 20 to 25–lbs.; which may sometimes rise to 34 or 35–lbs., and even to upwards of 50–lbs. during storms of great severity. Pressures much greater than 50–lbs. have been recorded, but they are wholly untrustworthy. Up to the present time, indeed, all wind pressure data are most unreliable, and to this fact may be attributed the frequent wide divergence of opinion as to the necessary wind allowance in any particular case. The great differences that exist in all recorded wind pressures are primarily due to the unphilosophic, unscientific, and unpractical character of the anemometers which give no correct information either as to pressure or velocity. The inertia of the moving parts, the transformation of velocities into pressures, and the injudicious placing of the anemometer, which renders it subject to local currents, all tend to vitiate the results.

It would be practically absurd to base calculations upon the violence of a wind-gust, a tornado, or other similar phenomena, as it is almost absolutely certain that a structure would not lie within its range In fact, it may be assumed that a wind pressure of 40 lbs. per sq. ft. upon a surface perpendicular to the direction of blow is an ample and perfectly safe allowance, especially when it is remembered that a greater pressure than this would cause the overthrow of nearly all the existing towers, chimneys, etc.

(4) *Wind pressure upon inclined surfaces.*—The pressure upon an inclined surface may be obtained from the following formula, which was experimentally deduced by Hutton, viz :

$$P_n = P.\,\sin a^{1.84 \sin a - 1} \qquad (A),$$

P being the intensity of the wind pressure in lbs. per sq. ft. upon a surface perpendicular to the direction of blow, and

P_n being the normal intensity upon a surface inclined at an angle a to the direction of blow.

Let P_h, P_v, be the components of P_n, parallel and perpendicular, respectively, to the direction of blow.

$$\therefore\ P_h = P_n.\,\sin a, \text{ and } P_v = P_n \cos a.$$

Hence, if the inclined surface is a roof, and if the wind blows horizontally, a is the roof's pitch.

Again, let v be the velocity of a fluid current in ft. per second, and be that due to a head of h–ft.

Let w be the weight of the fluid in lbs. per cubic ft.

Let p be the pressure of the current in lbs. per sq. ft. upon a surface perpendicular to its direction.

If the fluid, after striking the surface, is free to escape at right angles to its original direction,

$$\therefore\ p = 2.h.w = \frac{v^2}{g} \cdot w$$

Hence, for ordinary atmospheric air, since $w = .08$ lbs., approximately,

$$p = \frac{.08}{32} \cdot v^2 = \left(\frac{v}{20}\right)^2 \qquad \text{(B)}.$$

When the wind impinges upon a surface oblique to its direction, the intensity of the pressure is $\left(\frac{v.\sin\beta}{20}\right)^2$, v being the absolute impinging velocity, and β being the angle between the direction of blow and the surface impinged upon.

Tables prepared from formulæ A and B are given at the end of the chapter.

(5) *Distribution of Loads.*—Engineers have been accustomed to assume that the accidental load is uniformly distributed over the whole of the roof, and that it varies from 30 to 35–lbs. per sq. ft. of covered surface for short spans, and from 35 to 40–lbs. for spans of more than 60–ft. But the wind may blow on one side only, and although its direction is usually horizontal, it may occasionally be inclined at a considerable angle, and be even normal to a roof of high pitch. It is therefore evident that the horizontal component (p_h) of the normal pressure (p_n) should not be neglected, and it may cause a complete *reversal* of stress in members of the truss, especially if it is of the arched or braced type.

In the following examples it is assumed that the wind blows on one side only, and that the total load is concentrated at the panel points (or points of support) of the principal rafters.

Let w be the permanent load per sq. ft. of roof surface.

„ p_n „ normal wind pressure per sq. ft. of roof surface.

„ d „ distance in feet from centre to centre of principals.

„ l_1 and l_2 be the lengths in ft. of two adjacent panels.

The total *permanent* load at the panel point on the windward side

$$= w.\ d.\frac{l_1 + l_2}{2}$$

The total *accidental* load at the panel point on the windward side

$$= p_n .\ d.\ \frac{l_1 + l_2}{2}$$

The total load at the corresponding panel point on the leeward side

$$= w.\ d.\ \frac{l_1 + l_2}{2}$$

To determine the total stresses in the various members of the truss, two methods may be pursued, viz.;—

(*a*). The normal pressure (p_n) may be resolved into its vertical (p_v) and horizontal (p_h) components; p_v may be combined with the permanent load and p_h dealt with independently; the respective effects are to be added together.

(*b*). The permanent and accidental loads may be treated separately, and the respective effects added together.

If l is the length of a rafter, and if R' is the reaction at the windward support due to the horizontal pressure of the wind, by taking moments about the leeward support,

$$R'.2\ .\ l.\ \cos a + p_n .\ \sin a.\ l.\ d.\ \frac{l.\ \sin a}{2} = o, \text{ or } R' = -p_n . \frac{l.\ d}{4} . \frac{\sin^2 a}{\cos a}.$$

Hence, R' is *negative* and acts downwards; it must be less than the reaction at the same support due to the permanent load and the vertical wind pressure, or the roof will blow over.

Ex. 1.—Method (*a*) applied to the roof truss represented in Fig. 33.

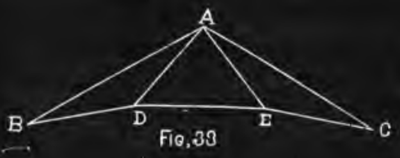


Fig. 33

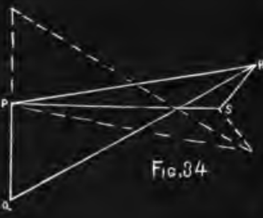


Fig. 34

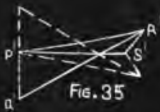


Fig. 35

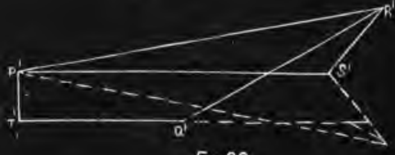


Fig. 36

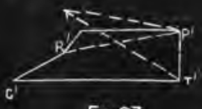


Fig. 37

Fig. 34 is the stress diagram for the vertical load upon the roof.

$p\,q$ is the vertical reaction at $B = \frac{l.\,d}{4}.\,(p_n.\ cos\,a + 2.\,w)$.

qr, pr, rs, ps, represent the stresses in the members to which they are respectively parallel, viz., the rafter AB, and the ties BD, DA, DE.

Fig. 35 is the stress diagram for the load due to the horizontal component of the wind pressure when there are no rollers under the heel of either of the rafters.

The horizontal reaction at each heel may be assumed to be one-half of the total horizontal force of the wind, and equal to $\frac{p_n.\,l.\,d}{2}.\,sin\ a$.

$p'\,q'$ is the *downward* reaction at $B = \frac{p_n\,l.\,d}{4}\cdot\frac{sin^2\,a}{cos\,a}$.

The stresses in the members AB, BD, DA, DE, are represented by the lines $q'r'$, $p'r'$, $r's'$, $p's'$, to which they are respectively parallel, and are evidently reversed in kind.

Thus the total resultant compression in $AB = qr - q'r'$,
the total resultant tension in $BD = pr - p'r'$, in $DA = rs - r's'$,
in $DE = ps - p's'$.

Fig. 36 is the stress diagram for the load due to the horizontal component of the wind pressure, when rollers are placed at B.

The total horizontal reaction is now at C, and equal to $p_n.\ l.\,d.\,sin\ a$.

$p'\ t'$ is the *downward* reaction $B = \frac{p_n.\ l.\ d}{4}.\ \frac{sin^2\ a}{cos\ a}$

$t'\ q'$ is the horizontal force of the wind at B.

$q'\ r'$, $p'\ r'$, $r'\ s'$, $p'\ s'$, are the stresses in AB, BD, DA, DE, respectively, and the resultant stresses in the different members are $q\ r - q'\ r'$ $p\ r - p'\ r'$, $r\ s - r'\ s'$, and $p\ s - p'\ s'$, as before.

Fig. 37 is the stress diagram for the load due to the horizontal component of the wind pressure, when rollers are placed at C.

The total horizontal reaction of the wind is now at B, and equal to $p_n.\ l.\ d.\ sin\ a$.

The horizontal force of the wind at B is $\frac{p_n.l.d}{2}.\ sin\,a$, so that the resultant horizontal force at B is $\frac{p_n.l.d}{2}\cdot\ sin\ a = t'\ q'$.

$p'\ t'$ is the *downward* reaction at $B = \frac{p_n\,l.\,d}{4}.\ \frac{sin^2\,a}{cos\,a}$

The resultant stresses are obtained as above.

The stress in DE equilibrates the stresses in DA, BD, and also those in EA, EC, so that the stresses in DA, DB are respectively equal to those in EA, EC.

The dotted lines in the diagrams represent the stresses in the members $A\ C$, $E\ C$, $E\ A$, to which they are respectively parallel.

Ex. 2.—Method (*a*) applied to the roof truss represented in Fig. 38.

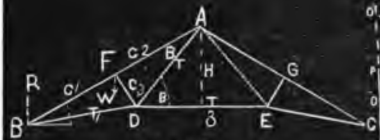


FIG. 38

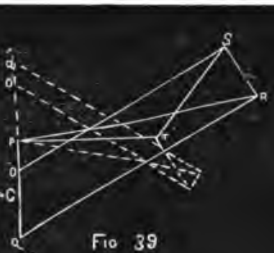


FIG. 39

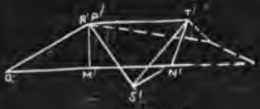


FIG. 40

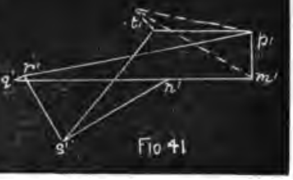


FIG. 41

Fig 39 is the stress diagram for the vertical load upon the roof.

pq is the vertical reaction at $B = \frac{l.\ d}{4}.\ (2.\ p_n.\ cos\ a + 3.\ w)$.

qo is the weight at $F = \frac{l.\ d}{4}.\ (2\ p_n.\ cos\ a + 2.\ w)$.

qr, os, sr, pr, st, pt, represent the stresses in the members to which they are respectively parallel, viz., BF, FA, DF, DB, DA, DE.

Fig. 40 is the stress diagram for the load due to the horizontal component of the wind pressure, when no rollers are placed either at B or C.

$p'm'$ is the downward reaction at $B = \frac{p_n.\ l.\ d}{4}.\frac{sin^2 a}{cos\ a}$.

$m'q'$ is the resultant horizontal force at $B = \frac{p_n.\ l.\ d}{4}.\ sin\ a$.

The line through q' parallel to the rafter BA, passes through p', so that there is no stress in BD from the horizontal wind pressure.

The stresses in the members BF, FA, DF, DA, DE, are represented by the lines, $q'r'$, $s'n'$, $p's'$, $s't'$, $p't'$, to which they are respectively parallel, and are evidently reversed in kind.

Thus, the total resultant compression in $BF = qr - q'r'$, in $FA = os - s'n'$, in $DF = sr - s'r'$, the total resultant tension in $BD = pr - o$, in $DA = st - s't'$, in $DE = pt - p't'$.

Fig 41 is the stress diagram for the load due to the horizontal component of the wind pressure when rollers are placed at B.

$p'm'$ is the vertical reaction at $B = \frac{p_n . l . d}{4} . \frac{\sin^2 a}{\cos a}$.

$m'q'$ is the resultant horizontal force at $B = \frac{3}{4} . p_n . l . d . \sin a$.

The total resultant stresses in BF, FA, DF, DB, DA, DE, are, $qr - q'r'$, $os - s'n'$, $sr - s'r'$, $pr - p'r'$, $st - s't'$, $pt - p't'$, respectively. The dotted lines in the diagrams represent the stresses in the members CG, GA, GE, EC, EA, to which they are respectively parallel.

pq', in Fig. 39, is the reaction at $C = \frac{l . d}{4} . (p_n . \cos a + 3 . w)$.

$q'o'$ is the weight at $G = \frac{1}{2} . l . d . w$.

Ex. 3.—Method (*a*) applied to the roof truss represented in Fig. 42.

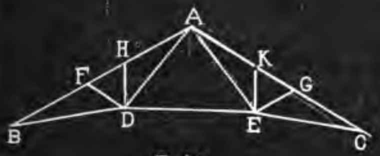


Fig 42

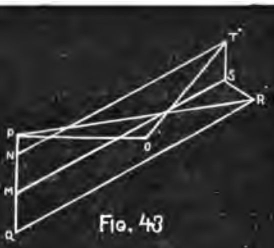


Fig. 43

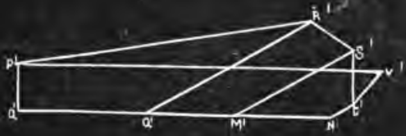


Fig. 44

Fig. 43 is the stress diagram due to the vertical load upon the roof.

pq is the vertical reaction at B

$$= \frac{p_n . \cos a}{4} . (3 . BH + AH) . d + \frac{w}{4} . (4 . BH + 2 . AH) . d.$$

qm is the weight at $F = \frac{p_n . \cos a + w}{2} . BH$ = the weight at $H = mn$

Fig. 44 is the stress diagram due to the horizontal component of the wind pressure, when rollers are placed at B.

$p'o'$ is the downward reaction at $B = \frac{p_n . l . d}{4} . \frac{\sin^2 a}{\cos a}$.

$o'q'$ is the horizontal force of the wind at $B = \frac{p_n . d . \sin a}{2} . BF$.

$q'm' = m'n'$, is the horizontal force of the wind at F or H, and is equal to $\frac{p_n . d . \sin a}{2} . BH$.

The total resultant stresses in the members BF, FH, HA, DF, DH, DB, DA, DE, are, $qr - q'r'$, $ms - m's'$, $nt - n't'$, $sr - s'r'$, $st - s't'$, $pr - p'r'$, $tv - t'v'$, $pv - p'v'$, respectively.

Ex. 4.—Method (a) applied to the roof truss represented in Fig. 45.

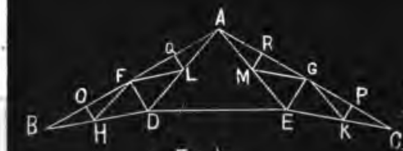


FIG 45

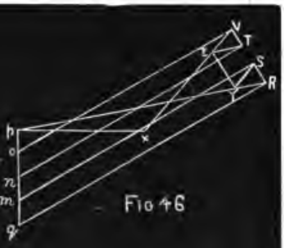


FIG 46

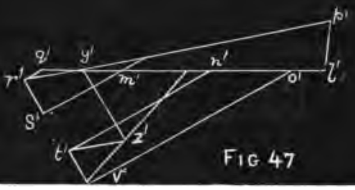


FIG 47

Fig. 46 is the stress diagram due to the vertical load upon the roof.

pq is the vertical reaction at $B = \frac{l . d}{16} . (10 . p_n . \cos a + 14 . w)$.

$qm = mn = no$, is the weight at O, F, or Q, and is equal to $\frac{l . d}{4} . (p_n . \cos a + w)$.

If $p_n . \cos a = w$, the point o coincides with p.

To avoid ambiguity it is generally assumed that the stresses in HO, HF, are equal to those in LQ, LF, respectively.

Fig. 47 is the stress diagram due to the horizontal component of the wind pressure, when rollers are placed at *C*.

$p'\,l'$ is the downward reaction at $B = \frac{p_n\, l.\, d}{4} \cdot \frac{\sin^2 a}{\cos a}$.

$l'\,q'$ is the resultant horizontal force at $B = \frac{7}{8} \cdot p_n.\, l.\, d.\, \sin a$.

$q'\,m' = m'\,n' = n'\,o'$, is the horizontal force of the wind at *O*, *F*, or *Q*, and is equal to $\frac{p_n.\, l.\, d}{4} .\ \sin a$.

Ex. 6.—Method (*a*) applied to the roof truss represented in Fig. 48.

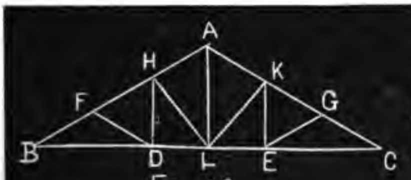


FIG 48

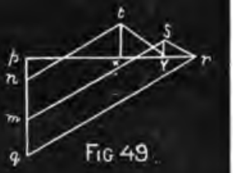


FIG 49

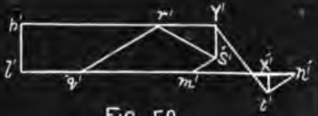


FIG 50

Fig. 49 is the stress diagram for the vertical load upon the roof.

pq is the vertical reaction at $B = \frac{l.\, d}{12} .\ (7.\, p_n \cos a + 10.\, w)$.

$qm = mn = \frac{l.\, d}{3} .\ (p_n.\, \cos a + w)$, is the weight at *F* or *H*.

Fig. 50 is the stress diagram due to the horizontal component of the wind pressure when rollers are placed at *B*.

$p'\,l'$ is the downward reaction at $B = \frac{p_n.\, l.\, d}{4} .\ \frac{\sin^2 a}{\cos a}$.

$l'\,q'$ is the horizontal force of the wind at $B = \frac{p_n.\, l.\, d}{6} .\ \sin a$.

Ex. 7.—Method (*b*) applied to the roof truss represented in Fig. 51.

Assuming the direction of the wind to be horizontal, the load at any joint of the bowstring roof in Fig. 51 due to wind may be taken to be the pressure upon a surface equal in area to the portion of the roof supported by one panel or bay, and inclined at the same angle to the horizontal as the tangent to the curve of the roof at the joint in question.

Let R be the resultant of the load P_1, P_2, P_3, P_4, at the joints B, d, e, f, and let it make an angle θ with the horizontal.

Fig. 52 is the stress diagram due to the normal wind pressure, when rollers are placed at C.

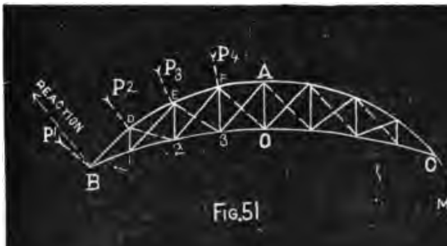


FIG. 51

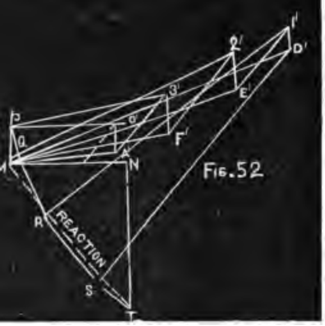


FIG. 52

The whole of the horizontal reaction ($= R. \cos \theta$) is at B, and is represented by mn.

nt is the vertical reaction at B.

ts, sr, rq, qp, represent the loads p_1, p_2, p_3, p_4, respectively.

sd', $r1'$ $q2'$, $p3'$, are the stresses in Bd, de, ef, fA "

md', me', mf', ma', " " $B1$, 12, 23, 30 "

$d'1'$, $e'2'$, $f'3'$, $a'o'$, " " $d1$, $e2$, $f3$, AO "

The stress diagrams for the normal wind pressure when rollers are placed at B, and also for the permanent load, may be easily drawn as already indicated.

(6).—*Analysis.* The results in the preceding examples may be easily verified analytically.

To determine, for example, the stresses in the members of the truss in Ex. 2 due to the vertical load.

Let R be the vertical reaction at $B = \frac{l.\ d}{4} \cdot (2\ p_n \cos a\ +\ 3.\ w)$. (1)

" W " " weight at $F = \frac{l.\ d}{4} \cdot (2\ p_n . \cos a + 2.\ w)$. (2)

" C_1, C_2, C_3, be the compressive stresses in BF, FA, DF, respectively.

" T_1, T_2, T_3, " " tensile " " DB, DA, DE " "

" the angle $ABD = \beta =$ the angle DAB ; $\therefore ADE = a + \beta$.

Resolve the forces at B perpendicular to the rafter,

$$\therefore R.\cos a = T_1.\sin\beta. \tag{3}$$

Resolve the forces at B perpendicular to the tie DB,

$$\therefore R.\cos\overline{a-\beta} = C_1.\sin\beta \tag{4}$$

Resolve the forces at F parallel to the rafter,

$$\therefore C_1 - W.\sin a = C_2 \tag{5}$$

Resolve the forces at F perpendicular to the rafter,

$$\therefore W.\cos a = C_3 \tag{6}$$

Resolve the forces at D perpendicular to the tie DE,

$$\left.\begin{aligned}\therefore T_1.\sin\overline{a-\beta} + C_3.\cos a &= T_2.\sin\overline{a+\beta}\\ &= R.\frac{\cos a}{\sin\beta}.\sin\overline{a-\beta} + C_3.\cos a. \text{ by (3)}\end{aligned}\right\} \tag{7}$$

Resolve the forces at D perpendicular to the tie DA,

$$\begin{aligned}\therefore T_1.\sin 2\beta - C_3.\cos\beta &= T_3.\sin\overline{a+\beta}\\ &= 2.R.\cos a.\cos\beta - C_3.\cos\beta, \text{ by (3)}\end{aligned} \tag{8}$$

From these equations, C_1, C_2, C_3, T_1, T_2, T_3, may be found in terms of p_n and w.

The value of T_3 may be verified by considering a section of the roof made by a plane a little on one side of A.

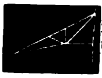
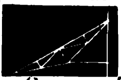
Let h be the distance of DE from A; take moments about A.

$$\therefore T_3.h = R.l.\cos a - W.\frac{l}{2}.\cos a$$

$$\text{or } T_3.\frac{l}{2}.\frac{\sin\overline{a+\beta}}{\cos\beta} = R.l.\cos a - W.\frac{l}{2}.\cos a$$

$\therefore T_3.\sin(a+\beta) = 2.R.\cos a.\cos\beta - W.\cos a.\cos\beta$, which agrees with 8, for $C_3 = W.\cos a$ by (6)

WEIGHTS OF VARIOUS ROOF FRAMINGS.

Description of Roof.	Location.	Cover-ing.	Span.	Width of Bays.	Weight in lbs per sq. ft. of covered area.		Pitch.
					Fram-ing.	Cover-ing.	
Pent	....	......	15′.0″		3.5		
Common Truss	....	......	37′.0″	5′.0″	4.6		
do	.. ..	......	40′.0″	12′.0″	5.5		
do	....	......	50′.0″	10′.0″	3.0		
do	Liverpool Docks	Felt	53′.3″	11′.0″	2.085	5.00	30°
Common Truss	....	......	54′.0″	14′.0″	9.5		
do	....	......	55′.0″	6′.6″	11.6		
do	....	......	72′.0″	20′.0″	7.0		
 Timber rafters and struts, iron ties.	Liverpool Docks	Zinc	62′.0″	12′.0″	3.013	5.66	30°
do “	do	Zinc	76′.0″	25′.0″	2.6	7.72	30°
do “	do	Zinc	79′.0″	13′.0″	3.86	5.42	30°
do “	do	Slates	80′.8″	11′.8″	4.72	12.1	26°.34′
 “ “	do		90′.2″	20′.0″	13.6		
Common Truss	....	......	84′.0″	9′.0″	8.5		
do	....	......	100′.0″	14′.0″	7.0		
do	....	......	130′.0″	26′.0″	6.4		
Bowstring	Manchester..	......	50′.0″	11′.0″	9.6		
do	Lime Street..	......	154′.0″	26′.0″	4.9		
do	Birmingham.	......	211′.0″	24′.0″	11.0		
Arched	Strasburg...	......	97′.0″	13′.0″	12.0		
do	Paris	......	153′.0″	26′.0″	15.0		
do	Dublin.......	......	41′.0″	16′.0″	10.7		
do	Derby	......	81′.6″	24′.0″	16.8		
do	Sydenham...	......	120′.0″		11.8		
do	do	......	72′.0″		11.3		
do	St. Pancras.	......	240′.0″	29′.4″	24.5		
do	Cremorne....	......	45′.0″	14′.6″	11.5		

WEIGHTS OF VARIOUS ROOF COVERINGS IN LBS. PER SQ. FT.

Thatch	6.5	Corrugated iron and laths	5.5
Felt, asphalted	.3 to .4	Slates, (ordinary)	5 to 9
Tin	.7 to 1.25	Slates, (large)	9 to 11
Sheet lead	5 to 8	Tiles	7 to 20
Sheet zinc	1.25 to 2.0	Pantiles	6 to 10
Copper	.8 to 1.25	Slates and iron laths	10.0
Sheet iron, (1-16 in. thick)	3.0	Boarding, ($\frac{3}{4}$-in. thick)	2.5
Sheet iron, (corrugated)	3.4	Boarding and sheet iron, (20 W. G.)	6.5
Cast-iron plates, ($\frac{3}{8}$-in. thick)	15.0	Timbering of tiled and slated roofs, additional	5.5 to 6.5
Sheet iron, (16 W.G.), and laths	5.0		

Table of the values of P_n, P_v, P_h, in lbs. per sq. ft. of surface, when $P = 40$, as determined by the formula $P_n = P. \sin a^{1.84 \sin a - 1}$

Pitch of roof.	P_n	P_v	P_h
5°	5.0	4.9	.4
10°	9.7	9.6	1.7
20°	18.1	17.0	6.2
30°	26.4	22.8	13.2
40°	33.3	25.5	21.4
50°	38.1	24.5	29.2
60°	40.0	20.0	34.0
70°	41.0	14.0	38.5
80°	40.4	7.0	39.8
90°	40.0	0.0	40.0

Table prepared from the formula, $p = \left(\frac{v}{20}\right)^2$.

Velocities in feet per second.	Velocities in miles per hour.	Pressure in lbs. per sq. ft.
10	6.8	.25
20	13.6	1.00
40	27.2	4.00
60	40.8	9.00
70	47.6	12.25
80	54.4	16.00
90	61.2	20.25
100	68.0	25.00
110	74.8	30.25
120	81.6	36.00
130	88.4	42.25
150	102.0	56.25

(1). If the pole of a funicular polygon describe a straight line, shew that the corresponding sides of successive funicular polygons with respect to successive positions of the pole will intersect in a straight line which is parallel to the locus of the pole.

(2). A system of heavy bars, freely articulated, is suspended from two fixed points; determine the magnitudes and directions of the stresses at the joints. If the bars are all of equal weight and length, shew that the tangents of the angles which successive bars make with the horizontal are in arithmetic progression.

(3). If an even number of bars of equal length and weight rest in equilibrium in the form of an arch, and if a_1, a_2,... a_n, be the respective angles of inclination to the horizon of the 1st, 2nd, ... n^{th} bars counting from the top, shew that $\tan a_{n+1} = \frac{2n+1}{2n-1} \cdot \tan a_n$.

(4). Four bars of equal weight and length, freely articulated at the extremities, form a square *ABCD*. The system rests in a vertical plane, the joint *A* being fixed, and the form of the square is preserved by means of a horizontal string connecting the joints *B* and *D*. If *W* be the weight of each bar, shew (*a*).–that the stress at *C* is horizontal and $= \frac{W}{2}$, (*b*).–that the stress on *BC* at *B* is $W \cdot \frac{\sqrt{5}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{1}{2}$, (*c*).–that the stress on *AB* at *B* is $W. \frac{\sqrt{13}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{3}{2}$, (*d*).–that the stress upon *AB* at *A* is $\frac{5}{2}. W$, (*e*).–that the tension of the string is $2. W$

(5). Five bars of equal length and weight, freely articulated at the extremities, form a regular pentagon *ABCDE*. The system rests in a vertical plane, the bar *CD* being fixed in a horizontal position, and the form of the pentagon being preserved by means of a string connecting the joints *B* and *E*. If the weight of each bar be *W*, shew that the tension of the string is $\frac{W}{2}.(\tan 54^\circ + 3. \tan. 18^\circ)$, and find the magnitudes and directions of the stresses at the joints.

(6). Six bars of equal length and weight ($= W$), freely articulated at the extremities, form a regular hexagon.

First, if the system hang in a vertical plane, the bar *AB* being fixed in a horizontal position, and the form of the hexagon being preserved by means of a string connecting the middle points of *AB* and *DE*, shew that, (*a*).–the tension of the string is $3W$, (*b*).–the stress at *C* is $\frac{W}{2. \sqrt{3}}$

and horizontal, (*c*).–the stress at D is $W.\sqrt{\frac{13}{12}}$ and makes with the vertical an angle $tan^{-1}\, 2\sqrt{3}$.

Second, if the system rest in a vertical plane, the bar DE being fixed in a horizontal position, and the form of the hexagon being preserved by means of a string connecting the joints C and F, shew that, (*a*).–the tension of the string is $W.\sqrt{3}$, (*b*).–the stress at C is $W.\sqrt{\frac{31}{3}}$ and makes with CB an angle $sin^{-1}\sqrt{\frac{3}{124}}$, (*c*).–the stress at B is $W.\sqrt{\frac{7}{12}}$ and makes with CD an angle $sin^{-1}\sqrt{\frac{3}{28}}$

Third, if the system hang in a vertical plane, the joint A being fixed, and the form of the hexagon being preserved by means of strings connecting A with the joints E, D, and C, shew that, (*a*).–the tension of each of the strings AE and AC is $W.\sqrt{3}$, (*b*).–the tension of the string AD is $2W$, and determine the magnitudes and directions of the stresses at the joints, assuming that the strings are connected with pins distinct from the bars.

(7). Shew that the stresses at C and F in the first case of Ex. 6 remain horizontal when the bars AF, FE, BC, CD, are replaced by any others which are all equally inclined to the horizon.

(8). An ordinary jib-crane (Fig. 7, Chap. I) is required to lift a weight of 10-tons at a horizontal distance of 6-ft. from the axis of the post. The post is a hollow cast-iron cylinder of 10-ins. external diar.; find its thickness, assuming the safe tensile and compressive stress to be 3-tons per sq. in.

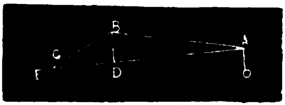
The hanging part of the chain is in *four* falls; the jib is 15-ft. long, and the top of the post is 12-ft. above ground; find the stresses in the jib and tie when the chain passes (1).–along the jib, (2).–along the tie.

The post turns round a vertical axis; find the direction and magnitude of the pressure at the toe, which is 3 ft. below ground.

(9). If the post in the preceding question were replaced by a solid cylindrical wrought-iron post, what should its diar. be; the safe inch-stress being 3 tons as before?

(10). The horizontal traces of the two back–stays of a derrick-crane are x and y ft. in length, and the angle between them is a; shew that the stress in the post is a maximum when $\frac{cos.(\beta-\theta)}{cos\,\theta} = \frac{x}{y}$, θ being the angle between the trace x and the plane of the jib and tie.

(11). The Fig. represents a crane employed in the construction of the staging for the Holyhead break-water. Its principal members are two wrought-iron rods ABC, supported at B upon two timber uprights BD, and connected at A and C with two timber longitudinals ADE. The crane rests upon a truck, and the



centre of pressure is maintained at D by means of a movable balance-box between D and E. The crane is required to lift a weight of 2-tons. determine the stresses in the different members. *(Data* :—$AE = 64$ ft., $AD = 39$-ft., $DC = 22$-ft., $BD = 8$-ft., vertical distance between A and $E = 7\frac{1}{6}$ ft.)

Point out a method by which the members may be effectively braced together.

(12). The inner flange of a bent crane, (Fig. 10, chap. I), forms a quadrant of a circle of 20-ft. radius, and is divided into *four* equal bays. The *outer* flange forms the segment of a circle of 23-ft. radius. The two flanges are 5-ft. apart at the foot, and are struck from centres in the same horizontal line. The bracing consists of a series of isosceles triangles, of which the bases are the equal bays of the inner flange. The crane is required to lift a weight of 10-tons, determine the stresses in all the members.

(13). A braced semi-arch is 10-ft. deep at the wall and projects 40-ft. The upper flange is horizontal, is divided into *four* equal bays and carries a uniformly distributed load of 40-tons. The lower flange forms the segment of a circle of 104-ft. radius. The bracing consists of a series of isosceles triangles of which the bases are the equal bays of the upper flange. Determine the stresses in all the members.

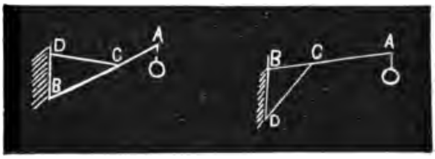
(14). Three bars, freely articulated, form an equilateral triangle ABC. The system rests in a vertical plane upon supports at B and C in the same horizontal line, and a weight W is suspended from A, determine the stress in BC, neglecting the weight of the bars.

(15). Three bars, freely articulated, form a triangle ABC, and the system is kept in equilibrium by three forces acting on the joints, determine the stress in each bar.

What relation holds between the stresses when the lines of action of the forces meet, (*a*).–in the centroid, (*b*).–in the orthocentre of the triangle ?

(16). A triangular truss of white pine consists of two equal rafters AB, AC, and a tie-beam BC; the span of the truss is 30-ft. and its rise is $7\frac{1}{2}$-ft.; the uniformly distributed load upon each rafter is 8400-lbs., and a weight of 10,000 lbs. is suspended from the centre of the tie-beam; determine the dimensions of the rafters and tie-beam, assuming the safe tensile and compressive inch stresses to be 3300 and 2700-lbs., respectively.

(17). The beam AB is supported at B and strengthened by a tie or strut CD; a weight W is suspended from A; shew how to determine the dimensions of the beam and brace.



(18). A triangular truss consists of two equal rafters *AB*, *AC*, and a tie-beam *BC*, all of white pine; the centre *D* of the tie-beam is supported from *A* by a wrought-iron rod *AD*; the uniformly distributed load upon each rafter is 8400-lbs. and upon the tie-beam is 36,000-lbs; determine the dimensions of the different members, *BC* being 40-ft. and *AD* 20-ft.

What will be the effect upon the several members if the centre of the tie-beam be supported upon a wall, and if for the rod a post be substituted against which the heads of the rafters can rest ?

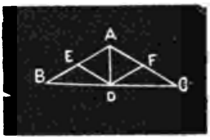
(19). A triangular truss of white pine consists of a rafter *AC*, a vertical post *AB*, and a horizontal tie-beam *AC*; the load upon the rafter is 300-lbs. per lineal ft.; $AC = 30$-ft., $AB = 6$-ft.; find the resultant pressure at *C*.

How much strength will be gained if the centre of the rafter be supported by a strut from *B* ?

(20). The rafters of a roof are 20-ft. long, and inclined to the vertical at 60°; the feet of the rafters are tied by two rods which meet under the vertex and are tied to it by a rod 5-ft. long; the roof is loaded with a weight of 3500-lbs. at the vertex, determine the stresses in all the members.

(21). The feet of the equal roof rafters *AB*, *AC*, are tied by rods *BD*, *CD*, which meet under the vertex and are joined to it by a rod *AD*. If *W* and *W'* are the distributed loads in lbs. upon the rafters, and if *s* is the span of the roof in ft., shew that the weight of metal in the ties in lbs. is $\frac{5}{6}.\frac{W+W'}{f}.s.\cot\beta$, *f* being the safe inch-stress in lbs., and β the angle *ABD*.

(22). The rafters *AB*, *AC*, in the Fig. are 20-ft. long, and are supported at the centres by the struts *DE*, *DF*; the centre *D* of the tie-beam *BC* is supported by a tie-rod *AD*, 10-ft. long; the uniformly distributed load upon *AB* is 8000-lbs., and upon *AC* is 2400-lbs.; determine the stresses in all the members.

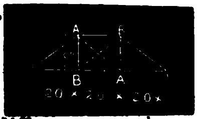


What will be the effect upon the several members if *AB* be subjected to a horizontal pressure of 156-lbs. per lineal ft.?

(23). Determine the dimensions of all the members of the truss in the preceding question, assuming the tie-beam to be also loaded with a weight of 1200-lbs. per lineal ft.

(24). A roof truss consists of two equal rafters inclined at 60° to the vertical, of a horizontal tie-beam of length l, of a collar-beam of length $\frac{l}{3}$, and of a queen-post at each end of the collar-beam; the truss is loaded with a weight of 2600-lbs. at the vertex, a weight of 4000-lbs. at one collar-beam joint, a weight of 1200-lbs. at the other, and a weight of

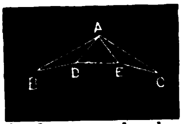
13,500-lbs. at the foot of each queen; determine the stress in each member.



(25). The platform of a bridge for a clear span of 60-ft. is carried by two timber trusses 15-ft. deep, and of the form shewn in the Fig.; the load upon the bridge is 50-lbs. per sq. ft. of platform, which is 12-ft. wide; what should be the dimensions of the different members, assuming the working stress to be 400-lbs. per sq. in. ?

If a single load of 6000-lbs. pass over the bridge, and if its effect be equally divided between the trusses, find the greatest stress produced in the diagonals *AA*, *BB*.

(26). A frame is composed of a horizontal top beam 40-ft. long, two vertical struts 3-ft. long, and three tie-rods of which the middle one is horizontal and 15-ft. long; find the greatest stress produced in the several members when a single load of 12,000-lbs. passes over the truss.



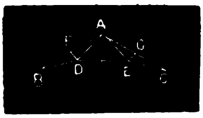
(27). The truss represented by the Fig. is composed of two equal rafters, *AB*, *AC*, four equal tie-rods *AD*, *BD*, *AE*, *CE*, and the horizontal tie *DE*.

If W be the distributed load upon *AB*, W' that upon *AC*, s the span, β the angle *ABD*, and f the safe inch-stress in the metal, shew that the total weight of the tie-rods is $\frac{5}{6}. \frac{W+W'}{f}. s. \cot \beta$.

(28). If the rafters in the preceding Question are inclined to the horizontal at an angle a, and if the tension on *BD* or *EC* is equal to that on *DE*, shew that $\beta = a$, (*i. e.*, *D* and *E* must fall in the horizontal line *BC*), or $\beta = 60^\circ - \frac{a}{3}$.

Determine the stress in *AD* corresponding to the latter value of β.

(29). The trusses for a roof of the same type as that in Question 27 are 12-ft. centre to centre, the span is 40-ft., the horizontal tie is 16-ft. long, the rafters are inclined at 60° to the vertical, the dead weight of the roof including snow, is estimated at 10-lbs. per sq. ft. of roof surface, there are no rollers either at *B* or *C*; determine the stress in each member when a wind blows horizontally on one side with a force of 40-lbs. per sq. ft. of vertical surface.



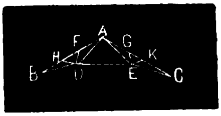
(30). The accompanying truss is similar to that in question 27 with the addition of the struts *DF*, *EG*.

(*a*).—With the same data as in question 27 shew that the weight of the metal in the tie rods is $\frac{5}{6}. \frac{s}{f}. \frac{W+(W+W^1). \cos^2\beta}{\sin \beta. \cos \beta}$,

and is a minimum when $\tan^2 \beta = 2 + \frac{W'}{W}$.

(*b*). If there are no rollers either at *B* or at *C*, and if a wind blows horizontally upon *AB*, shew that the horizontal component of its pressure upon *AB* produces no stress in the rod *BD*.

(*c*). In a given roof, the rafters are of pitch-pine, the tie rods of wrought-iron, the span is 60-ft., the trusses are 12-ft. centre to centre, *DF* = 5-ft. = *EG*, the angle *ABC* = 30°, the dead weight of the roof, including snow, is 9-lbs. per sq. ft. of roof surface, rollers are placed at *C*, a single weight of 3000-lbs. is suspended from *F*, and the roof is also designed to resist a normal wind pressure of 26.4-lbs. per sq. ft. of roof surface on one side; determine the dimensions of the several members.



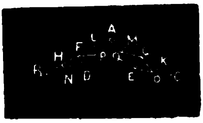
(31). The accompanying truss is similar to that in Question 27 with the addition of the *four* equal struts *DF*, *DH*, *EG*, *EK*.

(*a*).—With the same data as in question 27 shew that the weight of the metal in the tie-rods is

$$\frac{5}{18}\cdot\frac{s}{f}\cdot\frac{4\,W+3.\,(W+W').\ \cos^2\beta}{\cos\beta.\ \sin\beta},$$

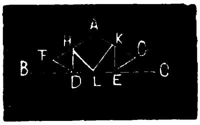
and is a minimum when $\tan^2\beta = \frac{7}{4}+\frac{3}{4}\cdot\frac{W'}{W}$.

(*b*).—In a truss of this type adopted for the roof of a shed at Birkenhead, the *rafters* are of pitch pine, are rectangular in section, have a constant breadth of 6½-ins., and a depth which gradually diminishes from 13-ins. at the heel to 9½-ins. at the peak; the *struts*, of which *DF* and *EG* are vertical, are also of pitch pine, are rectangular in section, and have a scantling of 8½-ins. by 6½-ins.; the *tie-rods* are of wrought-iron and *AD* = *BD* = *AE* = *CE* = 23–ft.; *BD* and *EC* are round 2¼-in. bars, *AD* and *AE* are round 1½-in. bars, *DE* is a round 1¾-in. bar; the angle *ABC* = 30°; the span = 79-ft.; the trusses are 13-ft. centre to centre; the heel *B* is free to slide on a smooth wall-plate; the dead weight of the roof, including snow, is 8-lbs. per sq. ft. of roof surface; determine the stress per unit of area to which each member is subjected when the wind blows horizontally with a force of 40-lbs. per sq. ft. of vertical surface, (1)—upon the side *AB*, (2)—upon the side *AC*.

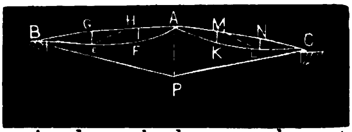


(32). The Fig. represents the form of truss recently adopted for the roof of a shed at Liverpool. The rafters are of pitch pine, are rectangular in section, have a depth of 14-ins., a breadth of 8-ins., are inclined at 60° to the vertical, and are each supported at 5 equidistant points; the six struts are also of pitch pine and rectangular in section; *DF* and *EG* are each 10-ft. 6-ins. long, and have a scantling of 12-ins. by 8-ins.; the remaining four struts have a scantlings of 8-ins. by 8-ins.; the tie-rods are of wrought-iron; *BN* and *CO* are round 2⅝-in. bars, *ND* and *OE* are round 2½-in. bars, *DP* and *EQ* are round 1⅞-in. bars, *PA* and *QA* are round 2¼-in. bars, *FN*, *FP*, *GQ*, and *GO* are round 1¼-in. bars, *DE* is a round 1¾-in. bar; the trusses are 20-ft. centre to centre, and the weight of a bay of the roof is 24,416-lbs; the span is 90 ft. 4-ins.; the heel *B* is free to slide upon a smooth wall-plate; determine the stress per unit of area in each member when the wind produces a normal pressure of 26.4-lbs. per sq. ft. of roof surface, (1)–upon the side *AB*, (2)–upon the side *AC*.

(33). Make an alternative design for the truss in the preceding question, using iron instead of timber for the rafters and struts.



(34). The trusses for a roof are to be 25-ft. centre to centre, and of the type represented by the Fig.; the span *BC* = 100-ft., the angle *ABC* = 30°, a single weight of 1000-lbs. is to be suspended from *H*, and one of 2000-lbs. from *G*, rollers are placed at *C*, the roof is to be designed to resist a horizontal wind pressure on one side of 40-lbs. per sq. ft. of vertical surface, the ties are to be of wrought-iron, the rafters and struts of timber or wrought-iron; determine the dimensions of the different members.



(35). The domed roof of a gas-holder for a clear span of 80 ft. is strengthened by secondary and primary trussing as in the Fig. The points *B* and *C* are connected by the tie *BPC* passing beneath the central strut *AP*, which is 15 ft. long, and is also common to all the primary trusses; the rise of *A* above the horizontal is 5 ft.; the secondary truss *ABEF*, consists of the equal bays *AH*, *HG*, *GB*, the ties *BE*, *EF*, *FA*, of which *BE* is horizontal and the struts *GE*, *FH*, which are each 2 ft. 6 in. long, and are parallel to the radius to the centre of *GH*; the secondary truss *ACLK* is similar to *ABEF*; when the holder is empty the weight supported by the truss is 36,000-lbs., which may be asumed to be concentrated at *G*, *H*, *A*, *M*, *N*, in the proportions, 8000, 4000, 1000, 4000 and 8000-lbs., respectively; determine the stresses in the different members of the truss.

Should the braces *HE* and *ML* be introduced? Why?

(36). Design the bowstring principals for the following roofs—

(*a*). Span = 50-ft., rise = 10-ft., depth of truss at centre = 5-ft., distance between principals = 11-ft.

(*b*). Span = 150-ft., rise = 30-ft., depth of truss at centre = 12-ft., distance between principals = 20-ft.

(*c*). Span = 200-ft., rise = 40-ft., depth of truss at centre = 20-ft., distance between principals = 20-ft.

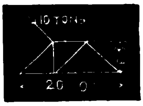
(*d*). Span = 100-ft., rise = 25-ft.

(37).—A funicular polygon is drawn for a series of parallel loads at given distances; shew that, (*a*).–by properly drawing the closing line of the polygon a bending moment curve is obtained which corresponds to any position of the series of loads on any given beam, (*b*).–so long as the closing line lies on the same two polygon sides, its positions for any given beam form the envelope of a parabola, (*c*).–the centre of the beam corresponding to a given closing line bisects the distance between the verticals through the intersection of the polygon sides and the point where the closing line touches the parabola.

(38). Vertical loads of 4, 3, 7 and 2-tons are concentrated upon a horizontal beam of 20-ft. span at distances of 3, 7, 12, and 15-ft., respectively, from the left support; prove generally that the vertical ordinate intercepted between a point on the corresponding funicular polygon and a closing line whose horizontal projection is the span of the beam, represents on a certain determined scale the bending moment of a section at the point; find its value by scale measurement for a section at 9-ft. from the left support, using the following scales :–for *lengths*, $\frac{1}{8}$-inch = 1-foot; for *forces*, $\frac{1}{2}$-inch = 1-ton; the polar distance = 5 tons.

Determine graphically by means of the same diagram the greatest bending moment that can be produced on the same section by the same series of loads travelling over the span at the stated distances apart.

(39). The inclined bars of the trapezoidal truss represented by the Fig. make angles of 45° with the horizontal; a load of 10 tons is applied at the top joint of the left rafter in a direction of 45° with the vertical; draw a frame diagram and determine graphically the two reactions, assuming the one at the right to be vertical; also, by means of a reciprocal figure, find the stresses in all the pieces of the frame.



Explain in what manner the right hand reaction may be made approximately vertical.

CHAPTER III.

BRIDGES.

(1).—*Classification.*—Bridges may be divided into three general classes, viz.; (1).—Bridges with horizontal girders, (2).—Suspension bridges, (3).—Arched bridges.

The present chapter will treat of bridges of the 1st class only. In these the platform is supported by two or more main girders resting upon two or more supports, and exerting thereon pressures which are vertical or very nearly so.

The neutral line (or surface) divides the girders into two parts, of which one is stretched and the other compressed.

(2).—*Comparative advantages of curved and horizontal flanges.*—Little, if any, advantage is gained by varying the depth of a girder, and the practice is open to many grave objections. The curved or parabolic form is not well suited to plate construction, and a diminution in depth lessens the resistance of the girder to distortion. Again, if the bottom flange is curved, the bracing for the lower part of the girder is restricted within narrow limits, and the girder itself must be independent, so that in a bridge of several spans any advantage which might be derivable from continuity is necessarily lost.

The depth is sometimes varied for the sake of appearance, but, generally speaking, the best and most economical form of girder is that in which the depth is uniform throughout, and in which the necessary thickness of flange at any point is obtained by increasing the number of plates.

(3).—*Depth of girder, or truss.*—The depth may vary from $\frac{1}{12}$th to $\frac{1}{7}$th of the span, and even more, the tendency at present being to approximate to the higher limit. If the depth fall below $\frac{1}{12}$th of the span, the deflection of the girder becomes a serious consideration.

The depth should not be more than about $1\frac{1}{2}$ times the width of the bridge, and is therefore limited to 24-ft. for a single and to 40-ft. for a double track bridge.

(4).—*Position of platform.*—The platform may be supported either at the top or bottom flanges, or in some intermediate position. It is claimed in favour of the last that the main girders may be braced together below the platform (Fig. 1), while the upper portions serve as parapets or guards, and also that the vibration communicated by a passing train is diminished. The position, however, is not conducive to rigidity, and a large amount of metal is required to form the connections.

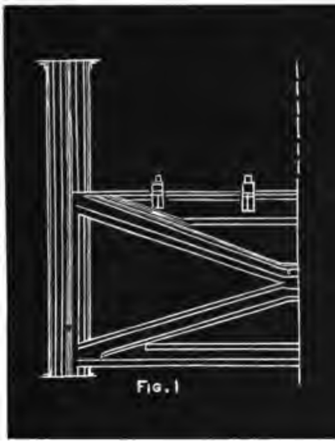


FIG. 1

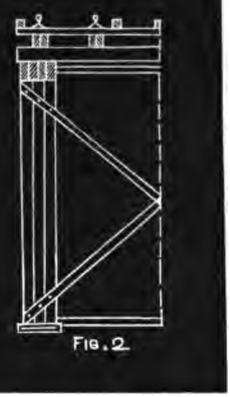


FIG. 2

The method of supporting the platform by the top flanges, (Fig. 2), renders the whole depth of the girder available for bracing, and is best adapted to girders of shallow depth. Heavy cross girders may be entirely dispensed with in the case of a single track bridge, and the load most effectively distributed, by laying the rails directly upon the flanges and vertically above the neutral line. Provision may be made for side spaces by employing sufficiently long cross-girders, or by means of short cantilevers fixed to the flanges, the advantage of the former arrangement being that it increases the resistance to lateral flexure, and gives the platform more elasticity.

Figs. 3, 4, 5, shew the cross girders attached to the bottom flanges, and the desirability of this mode of support increases with the depth of the main girders, of which the centres of gravity should be as low as possible. If the cross-girders are suspended by hangers or bolts below the flanges, (Fig. 5), the depth, and therefore the resistance to flexure, is increased.

In order to stiffen the main girders, braces and verticals, consisting of angle or tee-iron, are introduced and connected with the cross-girders by gusset-pieces, etc.; also, for the same purpose, the cross-girders may be prolonged on each side and the end joined to the top flanges by suitable bars.

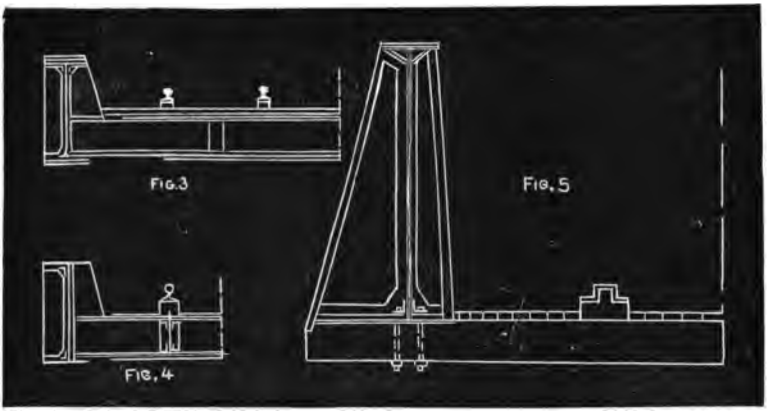


FIG. 3

FIG. 4

FIG. 5

When the depth of the main girders is more than about 5-ft., the top flanges should be braced together. But the minimum clear head-way over the rails is 16-ft., so that some other method should be adopted for the support of the platform, when the depth of the main girders is more than 5-ft. and less than 16-ft.

Assume that the depth of the platform below the flanges is 2-ft., and that the depth of the transverse bracing at the top is 1-ft.; the total *limiting* depths are 7-ft. and 19-ft., and if $\frac{1}{8}$ is taken as a mean ratio of the depth to the span, the corresponding limiting spans are 56-ft. and 152-ft.

(5).—*Comparative advantages of two, three and four main girders.*—A bridge is generally constructed with two main girders, but if it is crossed by a double track a third is occasionally added, and sometimes *each* track is carried by *two* independent girders.

The *two-girder* system, however, is to be preferred, as the rails, by such an arrangement, may be continued over the bridge without deviation at the approaches, and a large amount of material is economized, even taking into consideration the increased weight of long cross-girders.

The employment of *four* independent girders possesses the one great advantage of facilitating the maintenance of the bridge, as one-half may be closed for repairs without interrupting the traffic. On the other hand, the rails at the approaches must deviate from the main lines in order to enter the bridge, the width of the bridge is much increased, and far more material is required in its construction.

Few, if any, reasons can be urged in favour of the introduction of a third intermediate girder, since it presents all the objectionable features of the last system without any corresponding recommendation.

(6).—*Bridge loads.*—In order to determine the stresses in the different members of a bridge truss, or main girder, it is necessary to ascertain the amount and character of the load to which the bridge may be subjected. The load is partly *dead*, partly *live*, and depends upon the type of truss, the span, the number of tracks, and a variety of other conditions.

The *dead* load increases with the span, and embraces the weight of the main girders (or trusses), cross-girders, platform, rails, ballast, and accumulations of snow.

A table at the end of the chapter gives the dead load which may be adopted as a first approximation in calculating the strength and designing the parts of a bridge.

(7).—*Live load.*—The live load due to a passing train is by no means uniformly distributed, but consists of a number of different weights concentrated at definite points. This variable distribution is of little importance for spans of more than 100-ft., as the ratio of the dead to the live load then becomes very large, but it is of vital moment when the span is short.

The extreme weight of a locomotive upon its three drivers, with a wheel-base rarely exceeding 12-ft., varies from 72,000 to 84,000-lbs., and the maximum weight upon one pair of drivers may be taken at 30,000-lbs.

Thus, 30,000-lbs. may be concentrated at the centre of spans of 12-ft. and under. This is equivalent to 60,000-lbs., uniformly distributed, *i.e.*, to 12,000-lbs. for a 5-ft. span, and to 6000-lbs for a 10-ft. span, per lineal ft.

If the locomotive is placed centrally upon spans of 15, 20, 25, and 30-ft., and if it is assumed that its weight ($= 84{,}000$-lbs.) is equally distributed between the drivers, the bending moments at the centre are, 147,000, 252,000, 357,000, and 462,000 ft.-lbs., respectively. The equivalent uniformly distributed loads per lineal ft. are, $147{,}000 \times \frac{8}{15^2}$ ($= 5226\frac{2}{3}$-lbs.), $252{,}000 \times \frac{8}{20^2}$ ($= 5040$-lbs.), $357{,}000 \times \frac{8}{25^2}$ ($= 4569\frac{3}{5}$ lbs.), and $462{,}000 \times \frac{8}{30^2}$ ($= 4106\frac{2}{3}$-lbs.)

Two locomotives at the head of a train will cover a span of 60-ft., and the load is practically uniform, or 2800-lbs. per lineal ft.

Two locomotives and tenders will cover a span of 100-ft., and impose a practically uniform load of 2750-lbs. per lineal ft.

The coupling of more than two locomotives is sometimes necessary upon snow-roads, but rarely occurs elsewhere, and the case should not be included in a general rule for bridge design. It is a common practice, however, of the English Board of Trade, to prove a bridge by covering each track with as many locomotives and tenders as possible, and measuring the resulting deflection—an unreasonably severe proof for spans of more than 100-ft.

The weight of the heaviest train, exclusive of the engine, may be taken at 2250-lbs. per lineal ft. When a train crosses a bridge the consequent strains are far more intense than if the train were at rest, and in France the government regulations require a bridge to be proved under a moving as well as under a dead load. This additional intensity rapidly diminishes as the span increases, and may be disregarded for spans of more than 75-ft.

For spans of less than 75-ft. the live load may be reduced to an equivalent dead load by adding to the former certain arbitrary percentages, which may be taken at 10 for 75-ft., 20 for 60-ft., 30 for 50-ft., 40 for 40-ft., 50 for 30-ft., 60 for 20-ft., and 70 for 10-ft.

For spans of more than 75-ft. the *total load = dead load + live load.*

Provision may be made against accidents from high winds by the introduction of lateral bracing sufficiently strong to withstand a pressure of 40-lbs. per sq. ft. of truss and train.

Tables at the end of the chapter give the uniformly distributed live loads upon railway and highway bridges of different spans.

(8).—*Chord and web stresses.*—It may be assumed that the load upon a bridge is concentrated at the panel points of the main trusses.

The stresses produced in the chords (flanges) by the passage of a train are to be calculated on the basis of a uniformly distributed load.

In determining the strength of the web members allowance may be made for the irregularity in the distribution of the live load by substituting for the nearest standard panel weight a weight equal in amount to that upon an independent span of the same length as a panel.

(9).—*Bearing, Friction (also called expansion and bearing) rollers, etc.*—The area of the bearing surface at the supports (abutments) must be sufficient to prevent any undue crushing. The length of the bearing for a cast-iron girder is often made *one-fifteenth* of its central depth.

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In order to provide for the longitudinal contraction and expansion of a girder from changes of temperature, a smooth metallic sliding surface (bed-plate) is placed under one end, and, if the span is more than 60-ft., friction rollers are interposed between the girder and plate. The crushing strength of a cylindrical roller is proportional to the product of its length and diameter, and that of a sphere to the square of its diameter, while the relative strengths of a cube, the inscribed cylinder upon its base, the inscribed cylinder upon its side, and the inscribed sphere, are as, 100, 80, 32, 26. It is not desirable to subject cast-iron rollers to a greater pressure than that of 4000 to 6000-lbs. per lineal inch, or 340-lbs. per sq. in. of diametral section, and in practice, when there are several rollers, the working pressure should not exceed 185-lbs. per sq. in. of diametral section, as an allowance must be made for a variation in the position of the centre of pressure at the bearing, owing to the impossibility of loading the rollers equally.

The elastic strength of the rollers should remain unimpaired, and therefore the bed-plates should be fixed with the utmost care. Imperfect fixing cannot be remedied by increasing the number of the rollers.

It is uncertain whether cast-iron bed-plates continue long in working order, and hard-wood wall-plates are often substituted for them in spans of even 150-ft.

Let W be the *total* weight upon a bridge truss in lbs.
" B " breadth of the bearing at the roller end in inches.
" L " length " " " " " "

$$\therefore \frac{W}{2 \times 185} = B.L = \frac{W}{370}.$$

B for moderate spans usually exceeds the width of the flange (chord) by 5 or 6-in., and may be still further increased, in the case of large bridges.

NOTE.—In the construction of the Youghiogheny bridge, Pa., the specification requires that the pressure per lineal inch of roller is not to exceed the product of 500-lbs. by the square root of the diameter in inches ($500.\sqrt{d}$).

(10).—*Trellis, or Lattice Girders.*—The ordinary trellis or lattice girder consists of a pair of horizontal chords and two series of diagonals inclined in opposite directions (Fig. 6). The system of trellis is said to be single, double, treble,—according to the number of diagonals met by the same vertical section.

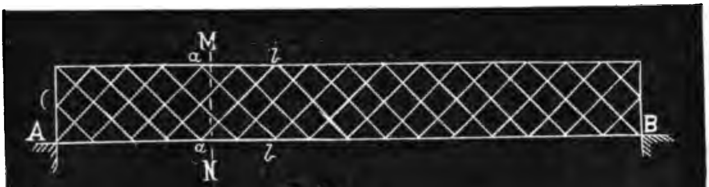


FIG 6

Vertical stiffeners, united to the chords and diagonals, may be introduced at regular intervals.

Figs. 7, 8, 9, 10 shew appropriate sections for the top chord; the bottom chord may be formed of fished and riveted plates, or of links and pins.

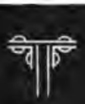


FIG. 7

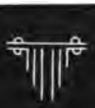


FIG. 8

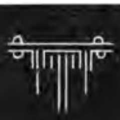


FIG. 9

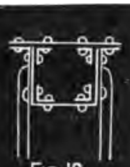


FIG. 10

The verticals and diagonals may be of an L, T, I, H, ⊔ or other suitable section, but the diagonals, except in the case of a single system of trellis, are usually flat bars, riveted together at the points of intersection.

An objection to this class of girder is the number of the joints.

The stresses in the diagonals are determined on the assumption that the shearing force at any vertical section is equally distributed between the diagonals met by that section, which is equivalent to the substitution of a *mean* stress for the different stresses in the several bars.

e. g., let w be the *permanent* load concentrated at each apex in Fig. 6.

Let θ be the inclination of the diagonals to the vertical.

The reaction at $A = 7\frac{1}{2}.\,w$, and the shearing force at the section $MN = 7\frac{1}{2}.\,w - 4.\,w = 3\frac{1}{2}.\,w$.

This shearing force must be transmitted through the diagonals.

Hence, the stress in ab due to the permanent load $= \frac{3\frac{1}{2}.\,w}{4}.\,sec\,\theta = \frac{7}{8}.\,w.\,sec\,\theta$.

Again, let w' be the *live* load concentrated at an apex.

The live load produces in ab the *greatest* stress of the *same* kind as that due to the permanent load, when it covers the longer segment up to and including b.

The shearing force at MN (= the reaction at A) is then $\frac{55}{16}.w'$, and the corresponding stress in ab is $\frac{55}{64}.w'.\sec\theta$.

$\therefore$ the maximum stress in ab due to both kinds of loads = $\left(\frac{7}{8}.w+\frac{55}{64}\,w'\right).\sec\theta$.

The shearing force produces its *greatest* effect in *reversing* the stress in ab due to the permanent load, when the live load covers the *shorter* segment up to and including a.

The shearing force at MN (= the reaction at B) is then $\frac{5}{8}.w'$, and the corresponding stress in ab is $\frac{5}{32}.w'.\sec\theta$;

$\therefore$ the resultant stress in ab when the live load covers the shorter segment $=\left(\frac{7}{8}.w-\frac{5}{32}.w'\right)\sec\theta$.

This stress may be negative, and must be provided for by introducing a counter-brace or by proportioning the bar to bear *both* the greatest tensile and the greatest compressive stress to which it may be subjected.

The stress in any other bar may be obtained as above.

The *chord* stresses are greatest when the live load covers the whole of the girder, and may be obtained by the method of moments, or in the manner described in the succeeding articles.

(11).—*Warren Girder.*—The Warren girder consists of two horizontal chords and a series of diagonal braces forming a single triangulation, or zig-zag, Fig. 11.

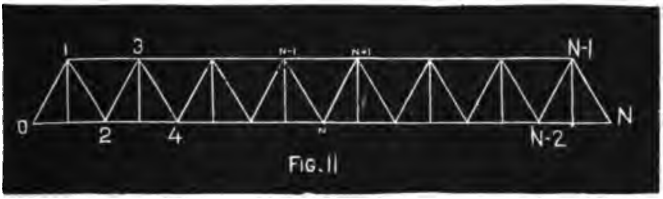


FIG. 11

The principles which regulate the construction of trellis girders are equally applicable to those of the Warren type.

The cross-girders (floor-beams) are spaced so as to occur at the apex of each triangle.

When the platform is supported at the top chords, the resistance of the structure to lateral flexure may be increased by horizontal bracing between the cross-girders and by diagonal bracing between the main-girders.

When the platform is supported at the bottom chords, additional cross-girders may be suspended from the apices in the upper chords, which also have the effect of adding to the rigidity of the main-girders.

Let w be the dead load concentrated at an apex, or joint.
" w' " " live " " " "
" l " " span of the girder.
" k " " depth " "
" s " " length of each diagonal brace.
" θ " " inclination " " " to the vertical
" N " " number of joints

$$\therefore sec\ \theta = \frac{s}{k},\ tan\ \theta = \frac{l}{N.k}.$$

Two cases will be considered.

CASE I.—All the joints loaded.

Chord Stresses.—These stresses are greatest when the live load covers the whole of the girder.

Let S_n be the shearing force at a vertical section between the joints n and $n+1$.

Let H_n be the horizontal chord stress between the joints n–1 and $n+1$.

The *total* load due to both dead and live loads $= (w + w'). \overline{N-1}$.

The reaction at each abutment due to this total load $= \frac{w + w'}{2}. \overline{N-1}$.

The shearing forces in the different bays are,

$$S_0 = \frac{w + w'}{2}. \overline{N-1}, \text{ between 0 and 1,}$$

$$S_1 = \frac{w + w'}{2}. \overline{N-3}, \quad \text{" 1 " 2,}$$

$$S_2 = \frac{w + w'}{2}. \overline{N-5}, \quad \text{" 2 " 3,}$$

$$S_3 = \frac{w + w'}{2}. \overline{N-7}, \quad \text{" 3 " 4.}$$

$$\text{and } S_n = \frac{w + w'}{2}. (N-2n-1)$$

The corresponding diagonal stresses are,

$$S_0.sec\ \theta,\ S_1.sec\ \theta, \ldots\ldots, S_n.sec\ \theta.$$

The last stresses multiplied by *sin* θ give the increments of the chord stresses at each joint. Thus,

$$H_1 = \text{tension in } 0\,2 = S_0.\ tan\ \theta = \frac{w+w'}{2}.\ N\text{-}1.\frac{l}{N.k}$$

$$= \frac{w+w'}{2}.\frac{l}{k}.\frac{N-1}{N}.$$

$$H_2 = \text{compression in } 13 = S_0.\ tan\ \theta + S_1.\ tan\ \theta$$

$$= \frac{w+w'}{2}.\frac{l}{k}.\frac{N-1}{N} + \frac{w+w'}{2}.\frac{l}{k}.\frac{N-3}{N} = \frac{w+w'}{2}.\frac{l}{k}.\frac{2.(N-2)}{N}$$

$$H_3 = \text{tension in } 24 = H_1 + S_1.\ tan\ \theta + S_2.\ tan\ \theta = \frac{w+w'}{2}.\frac{l}{k}.\frac{3.(N-3)}{N}$$

$$H_4 = \text{comp'n in } 35 = H_2 + S_2.\ tan\ \theta + S_3.\ tan\ \theta = \frac{w+w'}{2}.\frac{l}{k}.\frac{4.(N-4)}{N}$$

and H_n = horizontal stress in chord, between the joints n–1 and $n+1$

$= \frac{w+w'}{2}.\frac{l}{k}.\frac{n.\ N-n}{N}$, being a tension for a bay in the bottom chord, and a compression for a bay in the top chord.

Note.—The same results may be obtained by the method of moments. *e. g.*, find the chord stress between the joints $n-1$ and $n+1$.

Let a vertical plane divide the girder a little on the right of n.

The portion of the girder on the left of the secant plane is kept in equilibrium by the reaction at the left abutment, the horizontal stresses in the chords, and the stress in the diagonal n, $n+1$.

Take moments about the joint n,

$$\therefore H_n.\ k = \frac{w+w'}{2}.N\text{-}1.\ n.\frac{l}{N} - \overline{n\text{-}1}.\ \overline{w+w'}.\frac{n.\ l}{2.\ N}$$

$$= \frac{w+w'}{2}.\ l.\ n.\frac{N-n.}{N}$$

$$\therefore H_n = \&c.$$

Diagonal Stresses due to dead load.

Let d_n be the stress in the diagonal n, $n+1$, due to the dead load.

The shearing forces in the different bays due to the dead load are,

$\frac{w}{2}.N-1$, between 0 and 1, $\frac{w}{2}.N-3$, between 1 and 2

$\frac{w}{2}.N-5$, " 2 " 3, $\frac{w}{2}.N-7$, " 3 " 4

....

and $\frac{w}{2}.(N-2\,n-1)$, between n and $n+1$.

The corresponding diagonal stresses are,

$$\text{a compression } \frac{w}{2}.N-1.\ sec.\ \theta = \frac{w}{2}.N-1.\ \frac{s}{k} = d_0 \text{ in } 01,$$

$$\text{a tension } \frac{w}{2}.N-3.\ sec.\ \theta = \frac{w}{2}.N-3.\ \frac{s}{k} = d_1 \text{ in } 12,$$

$$\text{a compression } \frac{w}{2}.N-5.\ sec.\ \theta = \frac{w}{2}.N-5.\ \frac{s}{k} = d_2 \text{ in } 23,$$

....

and the stress in the n-th diagonal between n and $n+1$ is,

$$d_n = \frac{w}{2}.\ (N-2n-1).\ \frac{s}{k},$$

being a tension or compression according as the brace slopes down or up towards the centre.

Diagonal stresses due to live load.

The live load produces the *greatest* stress in any diagonal, $(n, n+1)$, of the *same* kind as that due to the dead load, when it covers the *longer* of the segments into which the diagonal divides the girder; represent this maximum stress by D_n.

The live load produces the *greatest* stress in any diagonal, $(n, n+1)$, of a kind *opposite* to that due to the dead load, when it covers the *shorter* of the segments into which the diagonal divides the girder; represent this maximum stress by D'_n.

The shearing force at any section due to the live load as it crosses the girder is the reaction at the end of the unloaded segment, and the corresponding diagonal stress is the product of this shearing force by $sec\ \theta$, or $\frac{s}{k}$.

The values of the different diagonal stresses are,

D_0 = compression in 0 1 when all the joints are loaded

$$= \frac{s}{k}.\frac{w'}{2}.\frac{N.N-1}{N}.$$

D_1 = tension in 1 2 when all the joints except one are loaded

$$= \frac{s}{k}.\frac{w'}{2}.\frac{N-1.\ N-2}{N}$$

D_2 = compression in 2 3 " " " 1 and 2 "

$$= \frac{s}{k}.\frac{w'}{2}.\frac{N-2.\ N-3}{N}$$

D_3 = tension in 3 4 " " " 1, 2, and 3 "

$$= \frac{s}{k}.\frac{w'}{2}.\frac{N-3.\ N-4}{N}$$

....

D_n = stress in n, $n+1$ when all the joints except 1, 2, 3 . . . and n loaded

$$= \frac{s}{k} \cdot \frac{w'}{2} \cdot \frac{N-n \, . \, N-n-1}{N}.$$

D'_0 = stress in 0 1 before the load comes upon the girder = 0

D'_1 = compression in 1 2 when the joint 1 is loaded $= \frac{s}{k} \cdot \frac{w'}{N} \cdot 1$

D'_2 = tension " 2 3 " joints 1 and 2 " $= \frac{s}{k} \cdot \frac{w'}{N} \cdot 3$

D'_3 = compression " 3 4 " " 1, 2, & 3 " $= \frac{s}{k} \cdot \frac{w'}{N} \cdot 6$

....

D'_n = stress in n, $n+1$, " " 1, 2, ... & n " $= \frac{s}{k} \cdot \frac{w'}{N} \cdot \frac{n \, . \, n+1}{2}$

The total maximum stress in the n-th diagonal of the *same* kind as that due to the dead load $= d_n + D_n$.

The resultant stress in the n-th diagonal when the load covers the shorter segment $= d_n - D'_n$.

This resultant stress is of the same kind as that due to the dead load so long as $d_n > D'_n$, and need not be considered since $d_n + D_n$ is the maximum stress of that kind.

If $D'_n > d_n$, it is necessary to provide for a stress in the given diagonal of a kind opposite to that due to $d_n + D_n$, and equal in amount to $D'_n - d_n$.

This is effected by counterbracing or by proportioning the bar to bear both the stresses $d_n + D_n$ and $D'_n - d_n$.

CASE II.—*Only joints denoted by even numbers loaded.*

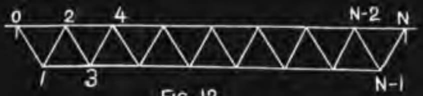


FIG. 12

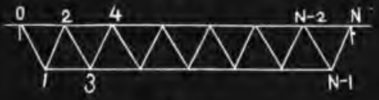


FIG. 13

Chord Stresses.—The stresses are greatest when the live load covers the whole of the girder.

The total load due to both dead and live loads $= (w + w').\frac{N-2}{2}.$

The reaction at each abutment due to this total load $= \frac{w + w'}{4}.\overline{N-2}.$

To find H_1. take moments about 1, $\therefore H_1.k = \frac{w + w'}{4}.\overline{N-2}.\frac{l}{N}$

" H_2. " " 2, $H_2.k = \frac{w + w'}{4}.\overline{N-2}.2.\frac{l}{N}.$

" H_3. " " 3, $H_3.k = \frac{w + w'}{4}.\overline{N-2}.3.\frac{l}{N}.$

$$- (w + w').\frac{l}{N}.$$

" H_4. " " 4, $H_4.k = \frac{w + w'}{4}.\overline{N-2}.4.\frac{l}{N}.$

$$- (w + w').2.\frac{l}{N}.$$

....

" H_n " " n, and *first* let n be even;

$$\therefore H_n.k = \frac{w + w'}{4}.\overline{N-2}.n.\frac{l}{N}$$

$$- (w + w').\frac{l}{N}.\{\overline{n-2} + \overline{n-4} + \ldots + 6 + 4 + 2\}$$

$$= (w + w').\frac{l}{N}.\left\{\frac{\overline{N-2}.n}{4} - \frac{n.\overline{n-2}}{4}\right\}$$

$$\text{and, } H_n = \frac{w + w'}{4}.\frac{l}{k}.\frac{n.\overline{N-n}}{N}.$$

Next let n be odd.

$$\therefore H_n.k = \frac{w + w'}{4}.\overline{N-2}.n.\frac{l}{N}$$

$$- (w + w').\frac{l}{N}.\{\overline{n-2} + \overline{n-4} + \ldots + 5 + 3 + 1\}$$

$$= (w + w').\frac{l}{N}.\left\{\frac{\overline{N-2}.n}{4} - \frac{\overline{n-1}^2}{4}\right\}$$

$$\text{and, } H_n = \frac{w + w'}{4}.\frac{l}{k}.\frac{\overline{N-2}.n - \overline{n-1}^2}{N}$$

Note.—If $\frac{N}{2}$ be even,

$\therefore H_{\frac{N}{2}}$, the stress in the middle bay $= \frac{w + w'}{16}.\frac{l}{k}.N$

If $\frac{N}{2}$ be odd,

$\therefore H_{\frac{N}{2}}$, the stress in the middle bay $= \frac{w + w'}{16} \cdot \frac{l}{k} \cdot \frac{N^2 - 4}{N}$.

Diagonal stresses due to the dead load.

The shearing forces in the different bays due to the dead load are, $\frac{w}{4} \cdot \overline{N-2}$ between 0 and 2, $\frac{w}{4} \cdot \overline{N-6}$ between 2 and 4, $\frac{w}{4} \cdot \overline{N-10}$ between 4 and 6, etc.

The corresponding diagonal stresses are,

$$d_0 \text{ in } 0\,1 = \frac{s}{k} \cdot \frac{w}{4} \cdot \overline{N-2} = d_1 \text{ in } 1\,2$$

$$d_2 \text{ in } 2\,3 = \frac{s}{k} \cdot \frac{w}{4} \cdot \overline{N-4} = d_3 \text{ in } 3\,4$$

$$d_4 \text{ in } 4\,5 = \frac{s}{k} \cdot \frac{w}{4} \cdot \overline{N-10} = d_5 \text{ in } 5\,6$$

etc., etc., etc.

Thus the stresses in the diagonals which meet at an unloaded joint are equal in magnitude but opposite in kind.

Diagonal stresses due to the live load.

These are found as in case I, and

$D_0 = D_1 \qquad D'_0 = D'_1$
$D_2 = D_3 \qquad D'_2 = D'_3$
$D_4 = D_5 \qquad D'_4 = D'_5$

If $\frac{N}{2}$ is odd, there is a single stress at the foot of each of these columns.

The maximum resultant stress due to both dead and live loads is obtained as before.

e. g., the maximum resultant stress in 3 4 when the longer segment is loaded $= d_2 + D_2 = d_3 + D_3$,

and the maximum resultant stress in 3 4 when the shorter segment is loaded $= d_2 - D'_2 = d_3 - D'_3$.

Note.— θ is generally 60°, in which case $s = 2 . \frac{l}{N}$.

(12).—*Howe Truss.*

Fig. 14 is a skeleton diagram of a Howe truss.

The truss may be of timber, of iron, or of timber and iron combined.

The chords of a timber truss usually consist of three or more parallel members, placed a little distance apart so as to allow iron suspenders with screwed ends to pass between them (Figs. 15, 16).

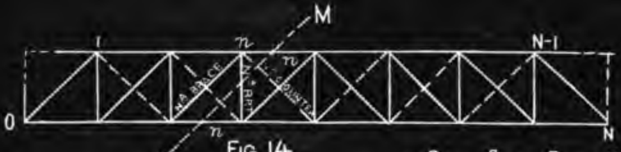


FIG. 14

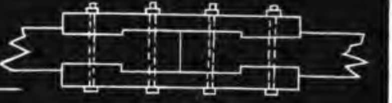


FIG. 17

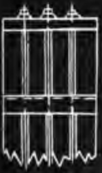


FIG. 15

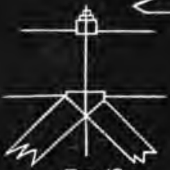


FIG. 16

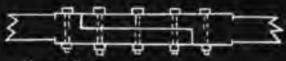


FIG. 18

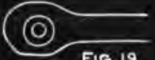


FIG. 19

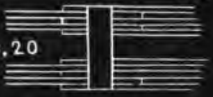


FIG. 20

Each member is made up of a number of lengths scarfed or fished together (Figs. 17, 18).

The main-braces, shewn by the full diagonal lines in Fig. 14 are composed of two or more members.

The counter-braces, which are introduced to withstand the effect of a live load, and are shewn by the dotted diagonal lines in Fig. 14, are either single or are composed of two or more members. They are set between the main braces, and are bolted to the latter at the points of intersection.

The main braces and counters abut against solid hard wood or hollow cast-iron angle-blocks (Fig. 16). They are designed to withstand compressive forces only, and are kept in place by tightening up the nuts at the heads of the suspenders.

The angle-blocks extend over the whole width of the chords; if they are made of iron, they may be strengthened by ribs.

If the bottom chord is of iron it may be constructed on the same principles as those employed for other iron girders. It often consists of a number of links, set on edge, and connected by pins (Figs. 19, 20).

In such a case the lower angle-blocks should have grooves to receive the bars, so as to prevent lateral flexure.

If the truss is made entirely of iron, the top chord may be formed of lengths of cast-iron provided with suitable flanges by which they can be bolted together. Angle-blocks may also be cast in the same piece with the chord.

To determine the stresses in the different members, the same data are assumed as for the Warren girder, except that N is now the number of panels.

Chord stresses.—These stresses are greatest when the live load covers the whole of the girder.

Let H_n be the chord stress in the n-th panel.

The total load due to both dead and live loads $= \overline{w + w'}. N-1$:

The reaction at each abutment due to this total load $= \frac{w + w'}{2}. \overline{N-1}$;

Let a plane MM' divide the truss as in Fig. 14. The portion of the truss on the left of the secant plane is kept in equilibrium by the load upon that portion, the reaction at the left abutment, the chord stresses in the n-th panels, and the tension in the n-th suspender.

First, let the load be on the top chord and take moments about the foot of the n-th suspender;

$$\therefore H_n. k = \frac{w + w'}{2}. N-1. n. \frac{l}{N} - \overline{w + w'}. \frac{l}{N}. \frac{n. n-1}{2}$$

$$= \frac{w+w'}{2}. l. \frac{n. N-n}{N}, \text{ or } H_n = \frac{w+w'}{2}. \frac{l}{k}. \frac{n. N-n}{N}.$$

Next let the load be on the bottom chord and take moments about the head of the n-th suspender.

$$\therefore H_n. k = \frac{w+w'}{2}. l. \frac{n. N-n}{N}, \text{ as before.}$$

Thus, H_n is the same for corresponding panels, whether the load is on the top or bottom chord.

Diagonal stresses due to the dead load:—

Let V'_n be the shearing force in the n-th panel, or the tension on the n-th suspender due to the dead load.

First, let the load be on the top chord,

$$\therefore V'_n = \frac{w}{2}. \overline{N-1} - n. w = w. \left(\frac{N-1}{2} - n\right).$$

Next, let the load be on the bottom chord,

$$\therefore V'_n = \frac{w}{2}. (N-1) - \overline{n-1}. w = w. \left(\frac{N+1}{2} - n\right);$$

The corresponding diagonal stresses are,

$$d_n = \frac{s}{k}.\ w.\left(\frac{N-1}{2} - n\right)$$

$$\text{and } d_n = \frac{s}{k}.\ w.\left(\frac{N+1}{2} - n\right)$$

Diagonal stresses due to the live load.

Let V''_n be the shearing force in the n-th panel, or tension on the n-th suspender, when the live load covers the longer segment.

First, let the load be on the *top* chord.

The greatest stress in the n-th brace of the same kind as that produced by the dead load occurs when all the panel points on the right of MM' are loaded.

$\therefore V''_n$, the shearing force on the left of MM' = the reaction at 0 $= \frac{w'}{2}.\ N-n-1.\ \frac{N-n}{N}$, and the corresponding diagonal stress

$$D_n = \frac{s}{k}.V''_n = \frac{s}{k}.\ \frac{w'}{2}.\ N-n-1.\ \frac{N-n}{N}$$

Hence, the resultant tension in the n-th suspender due to both dead and live loads $= V_n = V'_n + V''_n$

$$= w.\left(\frac{N-1}{2} - n\right) + \frac{w'}{2}.\ N-n-1.\frac{N-n}{N},$$

and the resultant maximum compression on the n-th brace due to both dead and live loads

$$= \frac{s}{k}.\ V_n = \frac{s}{k}.\ \left\{\ w.\left(\frac{N-1}{2} - n\right) + \frac{w'}{2}.\ \overline{N-n-1}.\frac{N-n}{N}\right\} = d_n + D_n$$

The live load tends to produce the greatest stress in the n-th counter when it covers the shorter segment up to and including the nth panel point. Even then there will be no stress in the counter unless the effect of the live load exceeds that of the dead load in the $(n+1)$-th brace.

The shearing force on the right of MM' = the reaction at $N = \frac{w'}{2}.\ \frac{n.n+1}{N}$.

D'_n the corresponding diagonal stress $= \frac{s}{k}.\ \frac{w'}{2}.\ \frac{n.\ n+1}{N}$

and the resultant stress in the counter $= D'_n - d_{n+1}$

$$= \frac{s}{k}.\left\{\frac{w'}{2}.\frac{n.\ n+1}{N} - w.\left(\frac{N-3}{2} - n\right)\right\}$$

Next let the load be on the *bottom* chord.

$$\therefore V''_n = \frac{w'}{2}.N-n.\ \frac{N-n+1}{N}$$

$$D_n = \frac{s}{k}.\ \frac{w'}{2}.\ N-n.\frac{N-n+1}{N}$$

Hence $V_n = V'_n + V''_n = w.\left(\frac{N+1}{2} - n\right) + \frac{w'}{2}. N-n. \frac{N-n+1}{N}$

and, $d_n + D_n = \frac{s}{k}.\left\{ w.\left(\frac{N+1}{2} - n\right) + \frac{w'}{2}. N-n. \frac{N-n+1}{N} \right\}$

Also the stress in the n-th counter is

$$D'_n - d_{n+1} = \frac{s}{k}.\left\{ \frac{w'}{2}. n. \frac{n+1}{N} - w.\left(\frac{N-1}{2} - n\right) \right\}$$

Note.—A common value of θ is 45°, when $sec\ \theta = \frac{s}{k} = 1.414$, and $\tan \theta = \frac{l}{N.k} = 1.$

The end panels and posts, shewn by the dotted lines in Fig. 14, may be omitted when the platform is suspended from the lower chords.

(13).—*Pratt, or Whipple truss.*

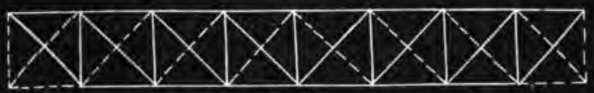


FIG. 21

Fig. 21 is a skeleton diagram of the simplest from of Pratt truss. The verticals are now in compression, and the braces in tension, so that the angle-blocks are placed above the top and below the bottom chord.

Counter-braces, shewn by the dotted diagonals in the Fig., are to be introduced if necessary, to withstand the effect of a live load.

The principles which regulate the construction of Howe trusses are equally applicable to those of the Pratt type.

The stresses in the different members are found precisely as in the preceding article.

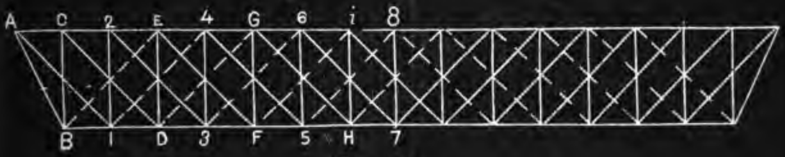


FIG. 22

Fig. 22 shews a modification of the Pratt truss, the braces forming two independent systems, which may be treated separately.

Let θ' be the inclination of AB to the vertical.
“ θ “ “ $A1$, CD, …… “

Chord stresses.—These stresses are greatest when the live load covers the whole of the girder.

The reaction at A from the system $ABCD$... $= 4.(w+w')$.
“ “ “ “ $a\,123$...... $= \frac{7}{2}.(w+w')$

The shearing forces in the different bays are,

$4.\ (w+w')$ in AC,	from the	system	$ABCD$...
$\frac{7}{2}.(w+w')$ in AC	“	“	$A1\ 2\ 3$...
$3.(w+w')$ in $C2$,	“	“	$ABCD$...
$\frac{5}{2}.(w+w')$ in $2E$	“	“	$A\,1\,2\,3$...
$2.(w+w')$ in $E4$,	“	“	$ABCD$...
$\frac{3}{2}.(w+w')$ in $4G$	“	“	$A\,I\,2\,3$...
$1.(w+w')$ in $G6$,	“	“	$ABCD$...
$\frac{1}{2}.(w+w')$ in $6i$	“	“	$A\,1\,2\,3$...

The corresponding diagonal stresses are,

$4.\ (w+w').\sec\theta'$ in AB, $3\frac{1}{2}.(w+w').\sec\theta$ in $A\,1$, $3.(w+w').\sec\theta$ in CD, $2\frac{1}{2}.\ (w+w').\sec\theta$ in 23, etc.

Hence the *top* chord stresses are,

C_1 in $AC = 4.(w+w').\tan\theta' + 3\frac{1}{2}.(w+w').\tan\theta$, C_2 in $C\,2 = C_1 + 3.\ (w+w').\ \tan\theta = 4.(w+w').\tan\theta' + 6\frac{1}{2}.(w+w').\tan\theta$, C_3 in $2E = C_2 + 2\frac{1}{2}.\ (w+w').\tan\theta = 4.(w+w').\tan\theta' + 9.(w+w').\ \tan\theta$, etc.

The *bottom* chord stresses are,

T_1 in $B\,1 = 4.(w+w').\tan\theta'$, T_2 in $1\ D = T_1 + 3\frac{1}{2}.\ (w+w').\tan\theta$
$= 4.(w+w').\tan\theta' + 3\frac{1}{2}.(w+w').\tan\theta = C_1$

So, $T_3 = C_2$, $T_4 = C_3$, etc., etc.

Again the stress in any diagonal 4 5 of the system $A\,1\,2$....due to the dead load $= 1\frac{1}{2}.w.\sec\theta$.

The live load produces the greatest stress in 45 of the same kind as that due to the dead load, when it is concentrated at all the panel points of the system $A\ 1\,2\,3$... on the right of 4.

The reaction at A is then $\frac{15}{8}.w'$,

and the corresponding diagonal stress $= \frac{15}{8}.w.'\sec\theta$.

∴ The maximum resultant stress in 4 5 $= \left(\frac{3}{2}.w + \frac{15}{8}.w'\right).sec\,\theta$

The live load tends to produce the greatest stress in any *counter* 5 8, when it is concentrated at all the panel points of the system *A* 1 2 3...... on the left of 8.

The reaction at the right abutment is then $\frac{3}{4}.w'$,

and the corresponding stress in the counter $= \frac{3}{4}.w'\,sec\,\theta$;

∴ the resultant stress in the counter $= \left(\frac{3}{4}.\,w' - \frac{1}{2}.\,w\right).sec\,\theta$,

$\frac{1}{2}.\,w.sec\,\theta$ being the stress in 6 7 due to the dead load.

Similarly the stresses in any other diagonal and counter may be found.

(14).—*Fink Truss.*

Fig. 23

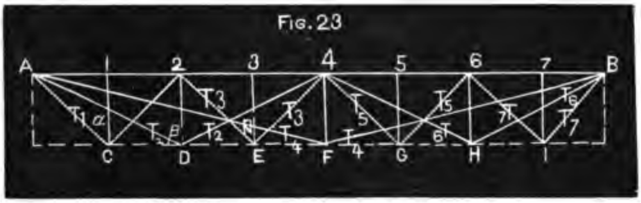


Fig. 23 represents a Fink truss. The platform may be either at the top or bottom.

The verticals are always struts and the obliques are always ties.

The stresses in the different members are to be calculated for a maximum distributed load.

Let weight upon post 1*C*, *i. e.*, ½ of weight between *A* and 2, be W_1
" " 2*D*, " " *A* " 4, be W_2
" " 3*E*, " " 2 " 4, be W_3
" " 4*F*, " " *A* " *B*, be W_4

and let W_5, W_6, W_7, be the weights upon the posts 5*G*, 6*H*, and 7*I*, respectively.

Let the inclinations of *AC*, *AD*, *AF*, to the vertical be α, β, γ, respectively.

Let T_1, T_2, T_3,...... be the tensions in the ties as in the Fig.

The tensions in the ties meeting at the foot of a post are evidently equal.

$$\begin{aligned}
\therefore\ 2.T_1.\cos a &= W_1 \\
2.T_2.\cos \beta &= W_2 + (T_1 + T_3).\cos a \\
2.T_3.\cos a &= W_3 \\
2.T_4.\cos \gamma &= W_4 + (T_2 + T_6).\cos \beta + (T_3 + T_5).\cos a \\
2.T_5.\cos a &= W_5 \\
2.T_6.\cos \beta &= W_6 + (T_5 + T_7).\cos a \\
2.T_7.\cos a &= W_7
\end{aligned}$$

$$\therefore\ T_1 = \frac{1}{2}.W_1.\sec a.,\ T_2 = \frac{1}{2}.\left(W_2 + \frac{W_1 + W_3}{2}\right).\sec \beta$$

$$T_3 = \frac{1}{2}.\ W_3.\ \sec a$$

$$T_4 = \frac{1}{2}.\left\{ W_4 + \frac{W_2 + W_6}{2} + \frac{W_1 + 3.W_3 + 3.W_5 + W_7}{4} \right\}.\sec \gamma$$

$$T_5 = \frac{1}{2}.\ W_5.\ \sec a,\ T_6 = \frac{1}{2}.\left(W_6 + \frac{W_5 + W_7}{2}\right).\sec \beta.,$$

$$\text{and } T_7 = \frac{1}{2}.\ W_7.\ \sec a.$$

Again the compression in $A2 = T_1 \sin a + T_2.\ \sin \beta + T_4.\ \sin \gamma$

" at $2 =$ $T_2.\ \sin \beta + T_4.\ \sin \gamma$

" in $24 =$ $T_2.\ \sin \beta + T_4.\ \sin \gamma + T_3 \sin a$

" at $4 =$ $T_4.\ \sin \gamma.$

etc. etc.

the compression in $1C = 2.T_1.\cos a = W_1$

" " $2D = 2.T_2.\cos \beta$

" " $3E = 2.T_3.\cos a = W_3$

" " $4G = 2.T_4.\cos \gamma$

etc. etc.

e. g., let the load upon the struss be a uniformly distributed load, W.

$$\therefore\ W_1 = W_3 = W_5 = W_7 = \frac{W}{8},\ W_2 = W_6 = \frac{W}{4},\ \text{and } W_4 = \frac{W}{2}.$$

$$\therefore\ T_1 = T_3 = T_5 = T_7 = \frac{W}{16}\cdot \sec a,\ T_2 = T_6 = \frac{3.W}{16}\cdot \sec \beta,\ \text{and } T_4 = \frac{W}{2}\cdot \sec \gamma.$$

If the truss represented by Fig. 23 is inverted, another system is obtained in which the obliques are struts and the verticals ties.

(15).—*Bollman Truss.*

Fig. 24 represents a Bollman truss. The platform may be either at the top or bottom.

The verticals are always struts and the obliques are always ties.

Each pair of ties, as *AC, BC,* is independent, and supports the load upon a vertical, *i. e.*, the weight of panel, or one-half of the weight between the panel points A and E.

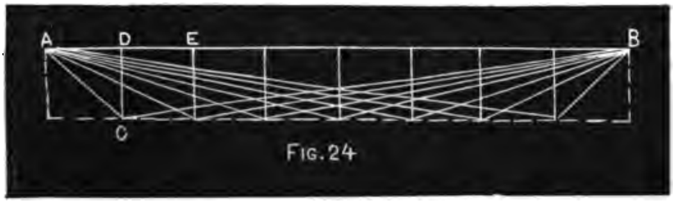


Fig. 24

Let T , T_2 be the stresses in AC, BC, respectively.

Let W_1 be the weight upon the post DC.

Let a_1, a_2, be the inclinations of AC, BC, respectively, to the vertical.

$$\therefore\ T_1 = W.\frac{\sin a_2}{\sin\,(a_1 + a_2)},\ \text{and}\ T_2 = W.\frac{\sin a_1}{\sin\,(a_1 + a_2)}$$

The stress in any other tie may be obtained in the same manner.

The compression in the top chord is the sum of the horizontal components of the stresses in all the ties which meet at one end.

(16).—*Quadrangular and Post systems.*—

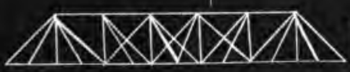


Fig 25

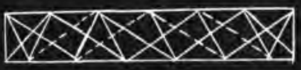


Fig 26

Fig. 25 represents the Quadrangular truss in which the bottom chord is supported at intermediate points half way between the verticals.

Fig. 26 represents the Post system. In reality it is the Pratt (Linville) truss shewn in Fig. 22, modified by inclining the compression members at an economical angle.

The stresses in the different members are to be calculated in the manner described in the preceding articles.

(17).—*Bowstring girder, or truss.*

The bowstring girder in its simplest form is represented by Fig. 27, and is an excellent structure in point of strength and economy.

The top chord is curved, and either springs from shoes (sockets) which are held together by a horizontal tie, or has its ends riveted to those of the tie.

The strongest bow is one composed of iron or steel cylindrical tubes, but any suitable form may be adopted, and the inverted trough offers special facilities for the attachment of verticals and diagonals.

The tie is constructed on the same principles as those employed for other iron girders, but in its best form it consists of flat bars set on edge and connected with the shoes by gibs and cotters.

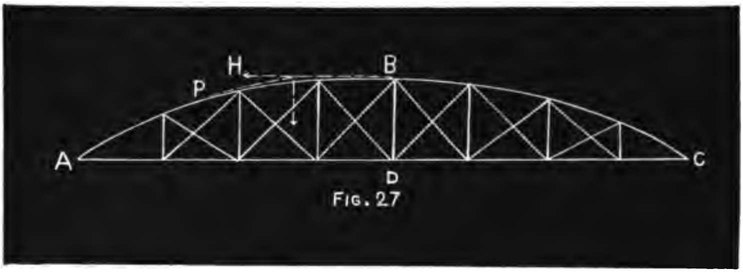


FIG. 27

The platform is suspended from the bow by means of vertical bars which are usually of an I section, and are set with the greatest breadth transverse, so as to increase the resistance to lateral flexure. In large bridges the webs of verticals and diagonals may be lattice-work.

If the load upon the girder is uniformly distributed and stationary, verticals only are required for its suspension, and the neutral axis of the bow should be a parabola. An irregularly distributed load, such as that due to a passing train, tends to change the shape of the bow, and diagonals are introduced to resist this tendency.

A circular arc is often used instead of a parabola.

To determine the stresses in the different members, assuming that the axis ABC of the top chord is a parabola :—

Let w be the dead load per lineal ft.
" w' " live " "
" l " span of the girder.
" k " greatest depth BD of the girder.

Chord stresses.—These stresses are greatest when the live load covers the whole of the girder.

The total load due to both dead and live loads $= (w + w').\ l$

The reaction at each abutment due to this total load $= \dfrac{w + w'}{2}.\ l$

Let H be the horizontal thrust at the crown.
Let T " " tension in the tie.

Imagine the girder to be cut by a vertical plane a little on the right of BD. The portion ABD is kept in equilibrium by the reaction at A the weight upon AD, and the forces H and T.

Take moments about B and D;

$$\therefore\ T.k = \frac{w+w'}{8}.l^2 = H.\,k,$$

$$\text{and } \therefore\ T = \frac{w+w'}{8}.\,\frac{l^2}{k} = H$$

Let H' be the thrust along the chord at any point P.

Let x be the horizontal distance of P from B.

The portion PB is kept in equilibrium by the thrust H at B, the thrust H' at P, and the weight $(w+w').x$ between P and B;

$$\therefore\ H^2.sec^2\,i = H'^2 = H^2 + (w+w)^2.x^2,$$

i being the inclination of the tangent at P to the horizontal.

$$\text{Hence, the thrust at } A = \left(\frac{w+w'}{2}.l\right)\left(\frac{l^2}{16.\,k^2}+1\right)^{1/2}$$

Diagonal stresses due to live load.

Assume that the load is concentrated at the panel points, and let it move from A towards C.

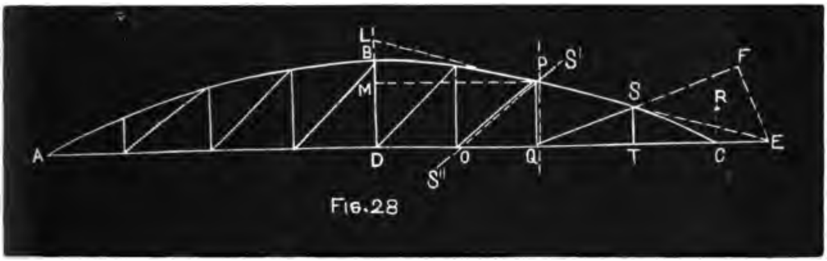


FIG. 28

If the diagonals slope as in the Fig. 28, they are all ties, and the live load produces the greatest stress in any one of them, as QS, when all the panel points from A up to and including Q are loaded.

Let x, y, be the horizontal and vertical co-ordinates, respectively, of any point on the parabola with respect to B as origin.

$$\text{The equation of the parabola is, } y = \frac{4.\,k}{l^2}.x^2 \qquad (1)$$

Let the tangent at the apex P meet DB produced in L, and DC produced in E.

Draw the horizontal line PM.

From the properties of the parabola, $LM = 2.\,BM$.

Let $PM = x$, and $BM = y$.

From the similar triangles LMP and LDE,

$$\frac{L\,M}{M\,P} = \frac{L\,D}{D\,E}, \text{ or } \frac{2.y}{x} = \frac{k+y}{x+QE}.$$

$$\therefore\ QE = x.\frac{k-y}{2.y} = \frac{l^2-4x^2}{8.x}$$

$$\text{Also, } CE = QE - \left(\frac{l}{2} - x\right) = \frac{(l-2x)^2}{8x}$$

$$\therefore\ \frac{CE}{QE} = \frac{l-2x}{l+2x} \tag{2}$$

Draw *EF* perpendicular to *QS* produced, and imagine the girder to be cut by a vertical plane a little on the right of *PQ*.

The portion of the girder between *PQ* and *C* is kept in equilibrium by the reaction *R* at *C*, the thrust in the bow at *P*, the tension in the tie at *Q*, and the stress in the diagonal *QS*.

Denote the stress by D_n, and let the panel *OQ* be the *n-th.*

Let θ be the inclination of *QS* to the horizontal.

Take moments about *E*,

$$\therefore\ D_n.\ EF = R.\ CE$$

$$\text{or } D_n = R.\frac{CE}{QE}.\ cosec\ . \tag{3}$$

Let *N* be the total number of panels,

$\therefore\ \frac{l}{N}$ is a panel length, and $w'.\frac{l}{N}$ is a panel weight.

$$\text{Also } x = n.\frac{l}{N} - \frac{l}{2}, \text{ and } \therefore \frac{l-2x}{l+2x} = \frac{CE}{QE} = \frac{N-n}{n}.$$

R, the reaction at *C* when the *n* panel points preceding *T* are loaded

$$= \frac{w'}{2}.\ l.\ \frac{n.\overline{n+1}}{N^2}$$

Hence, equation (3) becomes,

$$D_n = \frac{w'}{2}.l.\overline{n+1}.\frac{N-n}{N^2}.cosec\ \theta \tag{4}$$

Again, by equation (1), $k - ST = \frac{4.k}{l^2}.\ DT^2 = 4.k.\left(\frac{n+1}{N} - \frac{1}{2}\right)^2$

$$\therefore\ ST = k.\left\{1 - 4\left(\frac{n+1}{N} - \frac{1}{2}\right)^2\right\} = 4\,k.\frac{(N-n-1).(n+1)}{N^2}$$

$$\text{and } cosec\,\theta = \frac{QS}{ST} = \frac{(QT^2+ST^2)^{\frac{1}{2}}}{ST}$$

Thus, finally,

$$D_n = \frac{w'}{8}.\frac{l}{k}.\frac{N-n}{N}.\frac{\left[l^2 + \frac{16\,k^2}{N^2}.\{(N-n-1)\,(n+1)\}^2\right]^{\frac{1}{2}}}{N-n-1} \tag{5}$$

This formula evidently applies to all the diagonals between *B* and *C*.

Similarly, it may be easily shewn that the stress in any diagonal between B and A is given by an expression of precisely the same form.

Hence, the value of D_n in equation (5) is general for the whole girder.

A load moving from C towards A requires diagonals inclined in an opposite direction to those shewn in Fig. 28.

Stresses in the verticals due to the live load.

Let V_n be the stress in the n-th vertical PQ due to the live load. This stress is evidently a compression, and is a maximum when all the panel points from A up to and including O are loaded.

Imagine the girder to be cut by a plane $S' S''$ very near PO, Fig. 28. The portion of the girder between $S' S''$ and C is kept in equilibrium by the reaction R' at C, the thrust in the bow at P, the tension in the tie at O, and the compression V_n in the vertical.

Take moments about E.

$$V_n . QE = R'.CE., \text{ or } V_n = R'. \frac{N-n}{n},$$

and R', the reaction at C when the $(n-1)$ panel points from A up to and including O are loaded $= \frac{w'}{2}.l.\frac{n.\overline{n-1}}{N^2}$,

$$\text{Hence, } V_n = \frac{w'}{2}.l.\frac{(n-1).(N-n)}{N^2}, \qquad (6)$$

a general formula for all the verticals.

Let v_n be the tension in the n-th vertical due to the dead load. The resultant stress in it when the live load covers AO is $v_n - V_n$, and if negative, is the maximum compression to which PQ is subjected.

If $v_n - V_n$ is positive, the vertical PQ is never in compression.

The maximum tension in a vertical occurs when the live load covers the whole of the girder and $= w'.\frac{l}{N} +$ the tension due to the dead load.

Note.—The same results are obtained when N is odd.

(18).—*Bowstring girder with isosceles bracing.*

Diagonal stresses due to the dead load.—Under a dead load the bow is equilibrated, and the tie subjected to a uniform tensile stress equal in amount to the horizontal thrust at the crown. The braces merely serve to transmit the load to the bow and are all ties.

Let T_1, T_2 be the tensile stresses in the two diagonals meeting at any panel point Q. Let θ_1 θ_2 be the inclinations of the diagonals to the horizontal.

Let W be the panel weight suspended from Q.

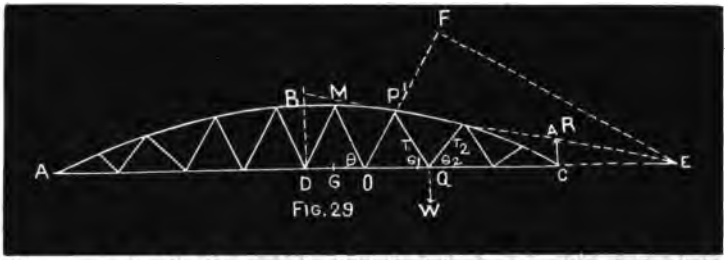


FIG. 29

The stress in the tie on each side of Q is the same, and therefore T_1, T_2, and W are necessarily in equilibrium.

$$\text{Hence, } T_1 = W.\frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}, \text{ and } T_1 = W.\frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}.$$

Diagonal stresses due to the live load.

Let N be the number of *half* panels.

The length of a panel $= \dfrac{2.l}{N}$; the weight at a panel point $= w'.\dfrac{2.l}{N}$.

Let the load move from A towards C. All the braces inclined like OP are ties, and all those inclined like QP are struts.
girder from A up to and including O.

The live load produces the greatest stress in OP when it covers the
Denote this stress by D_n; OG is the n-th half-panel.

$$\text{As before, } D_n.\,EF = R.\,CE. \qquad (1).$$

The load upon $AO = n.w'.\dfrac{l}{N}$, and $\therefore R = \dfrac{w'.n.l}{N}.\dfrac{n+2}{2.N}$

The ratio of CE to EF is denoted by the same expression as in the preceding article.

$$\therefore D_n = \frac{w'}{8}.\frac{l}{k}.\frac{n+2}{n+1}.\frac{N-n}{N}.\frac{\left[l^2+\dfrac{16\,k^2}{N^2}.\{(N-n-1)\,(n+1)\}^2\right]^{\frac{1}{2}}}{N-n-1} \qquad (2)$$

The live load produces the greatest stress in OM when it covers the girder up to and including D.

Denote this stress by D'_n; DG is now the n-th half panel.

Let R' be the reaction at C.

$$\text{As before, } D'_n = R'.\frac{CE}{OE}.\,\operatorname{cosec}\theta, \qquad (3)$$

θ being the angle MOD.

The weight upon $AD = (n-1).w'.\dfrac{l}{N}$, and $\therefore R' = \dfrac{w'}{2}.l.\dfrac{n^2-1}{N^2}$

It may be easily shown, as in the preceding article, that,

$$\frac{CE}{OE}=\frac{N-n-1}{n+1}, \text{ and } \operatorname{cosec}\theta = N.\frac{\left[l^2+\frac{16.k^2}{N^2}\{(N-n).n\}^2\right]^{\frac{1}{2}}}{4n.k.\ (N-n)}$$

$$\therefore D'_n=\frac{w'}{8}.\frac{l}{k}.\frac{n-1}{n}.\frac{N-n-1}{N}.\frac{\left[l^2+\frac{16\,k^2}{N^2}.\{(N-n).n\}^2\right]^{\frac{1}{2}}}{N-n} \qquad (4)$$

Hence, when the load moves from A towards C, equation (2) gives the diagonal stress when n is even, and equation (4) gives the stress when n is odd.

If the load moves from C towards A, the stresses are reversed in kind, so that the braces have to be designed to act both as struts and ties.

Note.—By inverting Fig. 29, a bowstring girder is obtained with the horizontal chord in compression and the bow in tension.

(19).—*Bowstring Suspension Bridge*, (*Lenticular Truss*).—This bridge is a combination of the ordinary and inverted bowstrings. The most important example is that erected at Saltash, Cornwall, which has a clear span of 445–ft. The bow is a wrought-iron tube of an elliptical section, stiffened at intervals by diaphragms, and the tie is a pair of chains.

A girder of this class may be made to resist the action of a passing load either by the stiffness of the bow, or by diagonal bracing.

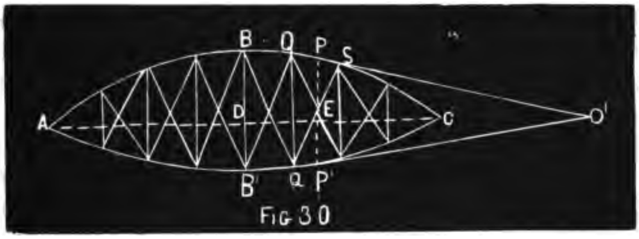


FIG 30

In Fig. 30, let $BD=k$, $B'D=k'$.

Let H be the horizontal thrust at B, and T the horizontal pull at B', when the live load covers the whole of the girder.

$$\therefore\ H=\frac{w+w'}{8}.\frac{l^2}{k+k'}=T.$$

First, let $k=k'$.

$$\therefore\ H=T=\frac{w+w'}{16}\frac{l^2}{k}, \text{ or one-half of the corres-}$$

ponding stress in a bow-string girder of span l and depth k.

One-half of the total load is supported by the bow and one-half is transmitted through the verticals to the tie.

$$\therefore \text{The stress in each vertical} = \frac{l}{N}.(w' + w''),$$

w'' being the portion of the dead weight per lineal ft. borne by the verticals, and N the number of panels.

The diagonals are strained only under a passing load.

Let PP' be a vertical through E, the point of intersection of any two diagonals in the same panel, and let the load move from A towards C.

By drawing the tangent at P and proceeding as in § (18), the expression for the diagonal stress in QS, becomes as before,

$$D_n = \frac{w'}{2}.\, l.\, \frac{n.(n-1)}{N^2}.\frac{l-2x}{l+2x}.\ cosec\ \theta. \tag{1}$$

Similarly the stress in the vertical QQ is

$$V_n = w''.\frac{l}{N}. + \frac{w'}{2}.\, l.\, \frac{n.(n-1)}{N^2}.\frac{l-2x}{l+2x}. \tag{2}$$

Next, let k and k' be unequal.

Let W be the weight of the bow, W' the weight of the tie.

$$\therefore \text{under these loads,} \frac{W}{8}\cdot\frac{l}{k} = H = H' = \frac{W'}{8}\frac{l}{k'}, \text{ or } \frac{W}{W'}, = \frac{k}{k'}. \tag{3}$$

The verticals are not strained unless the platform is attached to them along the common chord ADC. In such a case, the weight of the platform is to be included in W'.

The tangents at P and P' evidently meet AC produced in the same point O', for EO' is independent of k or k'. Hence, the stresses in the verticals and diagonals due to the passing load may be obtained as before.

(20).—*Camber.*—Owing to the play at the joints, a bridge truss, when first erected, will deflect to a much greater extent than is indicated by theory, and the material of the truss will receive a permanent set, which, however, will not prove detrimental to the stability of the structure, unless it is increased by subsequent loads.

If the chords were made straight they would curve downwards, and, although it does not necessarily follow that the strength of the truss would be sensibly impaired, the appearance would not be pleasing.

In practice it is usual to specify that the truss is to have such a *camber*, or upward convexity, that under ordinary loads the grade line will be true and straight.

The camber may be given to the truss by lengthening the upper or shortening the lower chord, and the *difference* of length should be equally divided amongst all the panels.

The lengths of the web members in a cambered truss are not the same as if the chords were horizontal, and must be carefully calculated, otherwise the several parts will not fit accurately together.

To find an approximate value for the Camber, etc.

Let d be the depth of the truss.

Let s_1, s_2, be the lengths of the upper and lower chords, respectively.

Let f_1, f_2, be the unit stresses in " " " "

Let d_1, d_2, be the distances of the neutral axis from the upper and lower chords respectively.

Let R be the radius of curvature of the neutral axis.

Let l be the span of the truss.

$$\therefore \frac{d_1}{R}=\frac{s_1-l}{l}=\frac{f_1}{E}, \text{ and} \frac{d_2}{R}=\frac{l-s_2}{l}=\frac{f_2}{E}, \textit{ approximately},$$

the chords being assumed to be circular arcs.

Hence, the excess in length of the upper over the lower chord

$$=s_1-s_2=\frac{l}{E}.(f_1+f_2).=l.\frac{d}{R}$$

Let x_1, x_2, be the cambers of the upper and lower chords, respectively.

$R+d_1$ and $R-d_2$ are the radii " " " "

By similar triangles,

the horizontal distance between the ends of the upper chord $=\frac{R+d_1}{R}.l$

" " " lower " $\frac{R-d_2}{R}.l$

Hence, $\left(\frac{1}{2}.\frac{R+d_1}{R}.l\right)^2=x_1.2.(R+d_1)$, approximately.

and $\left(\frac{1}{2}.\frac{R-d_2}{R}.l\right)^2=x_2.2.(R-d_2)$, approximately.

$$\therefore x_1=\frac{l^2}{8.R}\left(1+\frac{d_1}{R}\right), \text{ and } x_2=\frac{l^2}{8.R}\left(1-\frac{d_2}{R}\right).$$

TESTS FOR THE MATERIALS OF A BRIDGE.

I. WROUGHT IRON.

Tension Tests.—(1). All wrought-iron to have a limit of elasticity of not less than 26,000-lbs. per sq. in.

(2). Full sized bars of flat, round, or square iron, not over 4½-sq. ins. in sectional area to have an ultimate strength of 50,000-lbs. per sq. in., and to stretch 12½ per cent. of the whole length.

(3). Bars of a larger sectional area than 4½ sq. ins. to be allowed a reduction of 1,000-lbs. per sq. in. for each additional sq. in. of section down to a minimum of 46,000-lbs. per sq. in.

(4). Specimens of a uniform section of at least one sq. in., taken from bars of 4½-sq. ins. section and under, to have an ultimate strength of 52,000-lbs. per sq. in., and to stretch 18 per cent. in 8-inches.

(5). Similar specimens from bars of a larger section than 4½-sq. ins. to be allowed a reduction of 500-lbs. per sq. in. for each additional sq. in. of section, down to a minimum of 50,000-lbs. per sq. in.

(6). Similar specimens from angle and other shaped iron to have an ultimate strength of 50,000-lbs. per sq. in., and to stretch 15 per cent. in 8-inches.

(7). Similar specimens from plate-iron to have an ultimate strength of 48,000-lbs. per sq. in., and to stretch 15 per cent. in 8-inches.

Bending Tests.—(8). All iron for tension members to bend cold, without cracking, through an angle of 90° to a curve, of which the diameter is not more than *twice* the thickness of the piece, and at least one sample in three to bend 180° to this curve without cracking.

(9). Specimens from plate, angle, and other shaped iron, to bend cold without cracking, through an angle of 90° to a curve, of which the diameter is not more than *three* times the thickness of the specimen.

II. STEEL (MILD).

(1). The steel to have an elastic limit of 40,000-lbs. per sq. in., and a co-efficient of elasticity varying from 26,000,000 to 30,000,000-lbs.

(2). Full-sized bars to have an ultimate tensile strength of 70,000 lbs. per sq. in., and, when marked off in 12-in. lengths, to shew a stretch of 10 per cent. for the whole length, and of 15 per cent. at least for the length including the fracture.

(3). Bars drawn down to a suitable size from the crop ends of each bloom slab, and not more than one inch in thickness, to bend cold, without cracking, through an angle of 180° to a curves, of which the diameter is not more than the thickness of the bar.

III. CAST-IRON.

A specimen bar of the iron, 5-ft. long, 1-in. square, and resting upon supports 4-ft. 6-ins. apart, to bear, without breaking, a weight of 550-lbs. suspended at the centre.

Tables of the safe working loads upon compression members.

WROUGHT-IRON.

Ratio of length to least transverse dimension.	Safe working load in lbs. per sq. in.		Ratio of length to least transverse dimension.	Safe working load in lbs. per sq. in.	
	Flat ends.	Round ends.		Flat ends.	Round ends.
10	10,0000	7,000	30 " 35	6,000	4,000
10 to 15	9,000	6,500	35 " 40	5,000	3,500
15 " 20	8,000	6,000	40 " 50	3,800	2,500
20 " 25	7,500	5,500	50 " 60	3,000	2,000
25 " 30	6,800	5,000			

Cast-iron compression members should not exceed 22–diams. in length and should be subject to the same stresses as those prescribed for wrought-iron.

TIMBER.

Ratio of length to least transverse dimension.	Safe load in lbs. per sq. in.	
	OAK.	PINE.
10	1,000	900
10 to 20	800	700
20 " 30	600	500
30 " 40	400	300

RONDELET'S RULE FOR OAK AND PINE.

Ratio of length to least transverse dimension.	Extreme load in lbs. per sq. in. borne by pillar without lateral flexure.	Safe working load in lbs. per sq. in. with 7 as factor of safety.
1	5,974	853
12	4,978	711
24	2,987	426
36	1,991	284
48	996	142
60	498	71
72	249	35

In flexure, the greatest allowable stress per sq. in. may be taken at 1,200-lbs. for oak, and 1,000-lbs. for pine.

N. B.—The figures in the above tests and tables are by no means absolutely fixed, and are merely given to shew the standard practice of engineers.

Table of loads for Highway Bridges.

Span in feet.	City and Suburban Bridges liable to heavy traffic.	Bridges in Manufacturing Districts. Ballasted Roads.	Bridges in Country Districts. Unballasted Roads.
100 and under.	100 lbs. per sq. ft.	90 lbs. per sq. ft.	70 lbs. per sq. ft.
100 to 200	80 " "	60 " "	60 " "
200 to 300	70 " "	50 " "	50 " "
300 to 400	60 " "	50 " "	45 " "
400 and over...	50 " "	50 " "	45 " "

Table of loads for Railway Bridges.

Span in ft.	Dead load in lbs. per lineal ft.	Live load in lbs. per lineal ft.	Span in ft.	Dead load in lbs per lineal ft.	Live load in lbs. per lineal ft.
5	450	12,000	100	900	2750
10	500	5,500	125	1135	2600
15	550	5,000	150	1225	2500
20	625	4,750	175	1300	2500
25	625	4,500	200	1500	2400
30	650	4,000	250	2000	2400
40	700	3,500	300	2400	2250
50	750	3,250	350	3000	2250
75	800	3,000	400	4000	2250

CHAPTER IV.

SUSPENSION BRIDGES.

(1). *Cables.*—The modern suspension bridge consists of two or more cables, from which the platform is suspended by iron or steel rods. The cables pass over lofty supports (piers), and are secured to anchorages upon which they exert a direct pull.

Chain, or *link cables* are the most common in England and Europe, and consist of iron or steel links set on edge and pinned together. Formerly the links were made by welding the heads to a flat bar, but they are now invariably rolled in one piece, and the proportional dimensions of the head, which in the old bridges are very imperfect, have been much improved.

Hoop-iron cables have been used in a few cases, but the practice is now abandoned, on account of the difficulty attending the manufacture of endless hoop-iron.

Wire-rope cables are the most common in America, and form the strongest ties in proportion to their weight. They consist of a number of parallel *wire-ropes* or *strands*, compactly bound together in a cylindrical bundle by a wire wound round the outside. There are usually *seven* strands, one forming a core round which are placed the remaining *six*. It was found impossible to employ a *seven-strand* cable in the construction of the East River Bridge, as the individual strands would have been far too bulky to manipulate. The same objection held against a *thirteen-strand* cable (13 is the next number giving an approximately cylindrical shape), and it was finally decided to make the cable with *nineteen* strands. *Seven* of these are pressed together so as to form a centre core around which are placed the remaining *twelve*, the whole being continuously wrapped with wire.

In *laying up* a cable great care is required to distribute the tension uniformly amongst the wires. This may be effected either by giving each wire the same deflection or by using *straight* wire, *i.e.*, wire which when unrolled upon the floor from a coil remains straight, and shows no tendency to spring back. The distribution of stress is practically uniform in untwisted wire ropes. Such ropes are spun from the wires and strands without giving any twist to individual wires.

The *backstay* is the portion of the cable extending from an anchorage to the nearest pier.

The elevation of the cables should be sufficient to allow for *settling*, which chiefly arises from the deflection due to the load and from changes of temperature.

The cables may be protected from atmospheric influence by giving them a thorough coating of paint, oil or varnish, but wherever they are subject to saline influence, zinc seems to be the only certain safe-guard.

(2). *Anchorage, anchorage chains, saddles.*

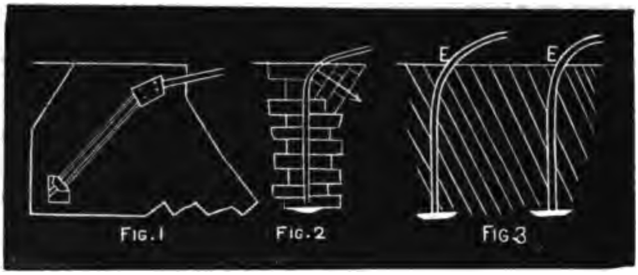


FIG. 1 FIG. 2 FIG. 3

The anchorage, or abutment, is a heavy mass of masonry or natural rock to which the end of a cable is made fast, and which resists by its dead weight the pull upon the cable.

The cable traverses the anchorage, as in Figs. 1, 2 and 3, passes through a strong, heavy, cast-iron anchor plate, and, if made of wire-rope, has its end effectively secured by turning it round a dead-eye and splicing it to itself. Much care, however, is required to prevent a wire-rope cable from rusting on account of the great extent of its surface, and it is considered advisable that the wire portion of the cable should always terminate at the entrance to the anchorage and there be attached to a massive chain of bars, which is continued to the anchor plate or plates and secured by bolts, wedges, or keys.

In order to reduce as much as possible the depth to which it is necessary to sink the anchor plates, the anchor chains are frequently curved as in Fig. 3. This gives rise to a force in a direction shewn by the arrows, and the masonry in the part of the abutment subjected to such force should be laid with its beds perpendicular to the line of thrust.

The anchor chains are made of compound links consisting alternately of an odd and even number of bars. The friction of the link-heads on the knuckle plates lessens to a considerable extent the stress in a chain, and it is therefore usual to diminish its sectional area gradually from the entrance E to the anchor. This is effected in the Niagara Suspension

Bridge by varying the section of the bars, and in the East River bridge by varying both the section and the number of the bars.

The necessity of preserving the anchor chains from rust is of such importance, that many engineers consider it most essential that the passages and channels containing the chains and fastenings should be accessible for periodical examination, painting and repairs. This is unnecessary if the chains are first chemically cleaned and then embedded in good hydraulic cement, as they will thus be perfectly protected from all atmospheric influence.

The direction of an anchor chain is changed by means of a saddle or knuckle plate, which should be capable of sliding to an extent sufficient to allow for the expansion and contraction of the chain. This may be accomplished without the aid of rollers by bedding the saddle upon a 4 or 5-in. thickness of asphalted felt.

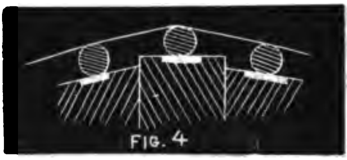


FIG. 4

The chain, where it passes over the piers, rests on *saddles*, the object of which is to furnish bearings with easy vertical curves. Either the saddle may be constructed as in Fig. 4, so as to allow the cable to slip over it with comparatively little friction, or the chain may be secured to the saddle and the saddle supported upon rollers which work over a perfectly true and horizontal bed formed by a saddle-plate fixed to the pier.

(3). *Suspenders.*—The suspenders are the vertical or inclined rods which carry the platform.

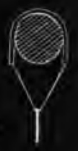


FIG. 5

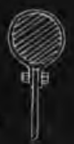


FIG. 6

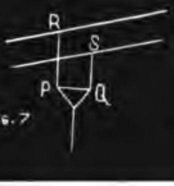


FIG. 7

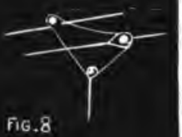


FIG. 8

In Fig. 5, the suspender rests in the groove of a cast-iron yoke which straddles the cable. Fig. 6 shews the suspender bolted to a wrought-iron or steel ring which embraces the cable. When there are more than two cables in the same vertical plane, various methods are adopted to ensure the uniform distribution of the load amongst the set. In Fig. 7, for example, the suspender is fastened to the centre of a small wrought-iron lever *PQ*, and the ends of the lever are connected with the cables by the equally strained rods *PR* and *QS*. In the Chelsea bridge, the distribution is made by means of an irregularly shaped plate (Fig. 8), one angle of which is supported by a joint pin, while a pin also passes through another angle and rests upon one of the chains.

The suspenders carry the ends of the cross-girders (floor-beams), and are spaced from 5 to 20-ft. apart. They should be provided with wrought-iron screw-boxes for purposes of adjustment.

(4).—*Curve of Cable; vertical suspenders.*—An arbitrarily loaded flexible cable takes the shape of one of the catenaries, but the *true* catenary is the curve in which a cable of uniform section and material hangs under its own weight only. If the load is uniformly distributed per horizontal unit of length, the cable forms a parabola. In most examples of suspension bridges the load is supported by rods from a certain number of points along each cable, and its distribution may be assumed to be approximately uniform per horizontal unit of length. Hence, the curve usually adopted for the cables of a suspension bridge is a parabola.

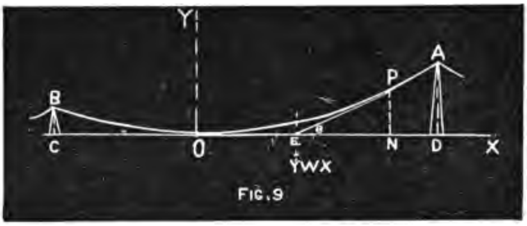


FIG. 9

Let *O* be the lowest point of the cable *AOB* passing over piers at *A* and *B*.

Let the load supported from the cable by *vertical* rods be *w per horizontal unit of length.*

Let $x\ y$ be the coordinates of any point *P* of the cable with respect to the horizontal *OX* and the vertical *OY* as axes of x and y, respectively.

Let θ be the inclination of the tangent at *P* to the horizontal.

The portion *OP* of the cable is kept in equilibrium by the horizontal pull *H* at *O*, by the tangential pull *T* at *P*, and by the load wx upon *OP* which acts vertically through the middle point *E* of *ON*, *PN* being the ordinate at *P*.

Hence, the tangent at *P* must also pass through *E*, and *PEN* is a triangle of forces.

$$\therefore \frac{H}{wx} = \frac{\frac{x}{2}}{y}, \text{ or } x^2 = \frac{2.H}{w} \cdot y, \qquad (1)$$

the equation to a parabola with its vertex at O, its axis vertical, and its parameter equal to $\frac{2.H}{w}$.

Again, $$\frac{T}{H} = \frac{PE}{EN} = \frac{1}{\cos\theta}, \quad \therefore H = T.\cos\theta, \qquad (2)$$

and the horizontal pull at every point of the cable is the same as that at the lowest point.

Also, $$\frac{T}{w.x} = \frac{PE}{PN} = \frac{\sqrt{y^2 + \frac{x^2}{4}}}{y}$$

or, $$T = w.x.\sqrt{1 + \frac{x^2}{4\,y^2}}. \qquad (3)$$

(5).—*Parameter, &c.*—Let h_1, h_2, be the elevations of A and B, respectively, above the horizontal line COD.

Let $OD = a_1$, $OC = a_2$, and let $a_1 + a_2 = a = CD$.

by equation (1), $a_1^2 = \frac{2.H}{w}.\,h_1$, and $a_2^2 = \frac{2.H}{w}.h_2$

$$\therefore \frac{a_1}{a_2} = \sqrt{\frac{h_1}{h_2}} \qquad (4)$$

But $a_1 + a_2 = a$,

$$\therefore a_1 = \frac{a.\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}, \text{ and } a_2 = \frac{a.\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \qquad (5)$$

$$\text{The Parameter} = \frac{2\,H}{w} = \frac{a_1^2}{h_1} = \frac{a^2}{h_1 + h_2 + 2.\sqrt{h_1 h_2}} \qquad (6)$$

$$\tan\theta = \frac{2.y}{x} = \frac{w.x}{H} = 2.\sqrt{y}.\frac{\sqrt{h_1} + \sqrt{h_2}}{a} \qquad (7)$$

Let θ_1, θ_2, be the values of θ at A and B, respectively.

$$\therefore \tan\theta_1 = 2.\sqrt{h_1}.\frac{\sqrt{h_1} + \sqrt{h_2}}{a}, \text{ and } \tan\theta_2 = 2.\sqrt{h_2}.\frac{\sqrt{h_1} + \sqrt{h_2}}{a} \qquad (8)$$

Corollary.—If $h_1 = h_2 = h$, $\therefore a_1 = a_2 = \frac{a}{2}$,

$$\text{the parameter} = \frac{a^2}{4.h}, \text{ by equation (6)}, \qquad (9)$$

$$\text{and } \tan\theta_1 = \tan\theta_2 = \frac{4.h}{a}, \text{ by equation (8)}. \qquad (10)$$

(6).—*Pressure upon piers, &c.*

Let T_1 be the tension in the main cable at A

" T' " " back-stay "

" θ' be the inclination to the horizontal of the tangent to the back-stay at A.

The total vertical pressure upon the pier $= T_1 . \sin\theta_1 + T' . \sin\theta' = P$

The resultant horizontal force at $A = T_1 \cos\theta_1 - T' . \cos\theta' = Q$

When the cable slides over smooth rounded saddles (Fig. 4), the tensions T_1 and T' are approximately the same;

$$\therefore P = T_1 . (\sin\theta_1 + \sin\theta'), \text{ and } Q = T_1 . (\cos\theta_1 - \cos\theta').$$

In order that the *resultant* pressure upon the pier may be wholly vertical Q must be zero, or $\theta_1 = \theta'$, and the corresponding pressure is,

$$P = 2 . T_1 . \sin\theta_1$$

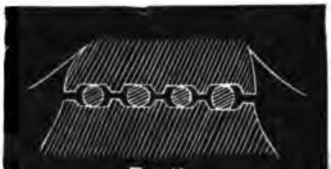


FIG. 10

Again, the resultant pressure may be made to act vertically, by securing the cable to a saddle which is free to move horizontally on the top of the pier, (Fig. 10).

In this case, $T_1 . \cos\theta_1 = T' . \cos\theta' . = H$

and $\therefore P = H . (\tan\theta_1 + \tan\theta')$.

Suppose the friction at the saddle too great to be disregarded.

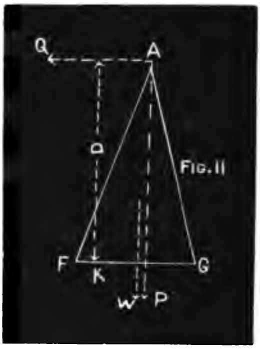


FIG. 11

Let D be the total height of the pier, and let W be its weight.

Let FG be the base of the pier, and K the limiting position of the centre of pressure.

Let p, q be the distance of P and W, respectively, from K;

$\therefore$ for *stability of position*, $Q \leqq \dfrac{P.p + W.q}{D}$

and for *stability of friction* when the pier is of masonry, $\dfrac{Q}{P + W} \leqq$ the co-efficient of friction.

The piers are made of timber, iron, steel, or masonry, and allow of great scope in architectural design.

The cable should in no case be rigidly attached to the pier, unless the lower end of the latter is free to revolve through a small angle about a horizontal axis.

(7).—*Length of arc of cable.*—Let $OP = s$.

$$\tan\theta = \frac{w.x}{H}, \text{ by equation (7),}$$

$$\therefore \sec^2 . \theta . d\theta = \frac{w}{H} . dx = \frac{w}{H} . ds . \cos\theta$$

$$\text{or } ds = \frac{H}{w} . \frac{d\theta}{\cos^3\theta} .$$

$$\text{Hence, } s = \frac{H}{w}.\int_0^\theta \frac{d\theta}{\cos^3\theta} = \frac{H}{2.w}.\{\tan\theta.\sec\theta + \log_e(\tan\theta + \sec\theta)\} \quad (11)$$

$$\text{Again, } \tan\theta = \frac{w}{H}.x, \text{ and } \sec\theta = \sqrt{1 + \frac{w^2}{H^2}.x^2}$$

$$\therefore\ s = \frac{H}{2.w}.\left\{\frac{w}{H}.x.\sqrt{1 + \frac{w^2}{H^2}.x^2} + \log_e\left\{\frac{w}{H}.x + \sqrt{1 + \frac{w^2}{H^2}.x^2}\right\}\right. \quad (12)$$

Corollary.—The length of any arc of the parabola measured from O is approximately the same as that of the osculatory circle at O measured from O and having the same *abscissa*.

Let R be the radius of this circle.

$$\therefore R = \frac{Parameter}{2} = \frac{x^2}{2.y}$$

$$\text{and} \frac{ds}{dx} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} = \left(1 + \frac{x^2}{R^2}\right)^{\frac{1}{2}} = 1 + \frac{x^2}{2.R^2}, \text{ approximately.}$$

$$\therefore\ s = x + \frac{x^3}{6.R^2} = x + \frac{2}{3}.\frac{y^2}{x} \quad (13)$$

(8).—***Weight of Cable.***—The ultimate tenacity of iron wire is 90,000 lbs. per sq. in., while that of steel rises to 160,000-lbs. and even more. The strength and gauge of cable wire may be ensured by specifying that the wire is to have a certain ultimate tenacity and elastic limit, and that a given number of lineal ft. of the wire is to weigh *one* pound. Each of the wires for the cables of the East River bridge was to have an ultimate tenacity of 3,400-lbs., an elastic limit of 1600-lbs., and 14 lineal ft. of the wire were to weigh *one* pound. A very uniform wire, having a coefficient of elasticity of 29,000,000-lbs., has been the result, and the process of *straightening* has raised the ultimate tenacity and elastic limit nearly 8 per cent.

Let W be the weight of a length a_1 $(= OD)$ of a cable of sufficient sectional area to bear safely the horizontal tension H.

Let W_2 be the weight of the length s_1 $(= OA)$ of the cable of a sectional area sufficient to bear safely the tension T_1 at A.

Let f be the safe inch stress.

Let p be the specific weight of the cable material.

$$\therefore\ W_1 = \frac{H}{f}.a_1.p, \text{ and } W_2 = \frac{H.\sec\theta_1}{f}.s_1.p$$

$$\therefore\ W_2 = W_1.\frac{s_1}{a_1}.\sec\theta_1 = \frac{W_1}{a_1}.\left(a_1 + \frac{2}{3}.\frac{h_1^2}{a_1}\right)\left(1 + \frac{2.h_1^2}{a_1^2} + \ldots\right)$$

$$\text{or } W_2 = W_1\left(1 + \frac{8}{3}.\frac{h_1^2}{a_1^2}\right), \text{ nearly.} \quad (14)$$

A saving may be effected by proportioning any given section to the pull across that section.

At any point (x, y), the pull$=H. sec\,\theta$, and the corresponding sectional area$=\frac{H.sec\,\theta}{f}$. The weight per unit of length$=\frac{H.sec\,\theta}{f}.p$, and the total weight of the length s_1 $(=OA)$ is,

$$W_2=\int_0^{a_1}\frac{H.sec\,\theta}{f}.p.\frac{ds}{dx}.\,dx=\frac{H.p}{f}\int_0^{a_1}sec^2\,\theta.dx=\frac{H.p}{f}.\int_0^{a_1}\left(1+\frac{2.y^2}{x^2}+\ldots\right)^2\cdot dx$$

$$\text{But } x^2=\frac{a_1^2}{h_1}.y$$

$$\therefore W_2=\frac{H.p}{f}.\int_0^{a_1}\left(1+2.\frac{h_1^2}{a_1^4}.x^2+\ldots\right)^2.dx=\frac{H.p}{f}.\left(a_1+\frac{4}{3}.\frac{h_1^2}{a_1}\right),\text{ nearly}$$

$$\text{Hence, } W_2=W_1.\left(1+\frac{4}{3}.\frac{h_1^2}{a_1^2}\right) \tag{15}$$

The weight of a cubic inch of steel averages .283-lb.
" " wrought iron " .277-lb.

The volume in inches of the cable of weight $W_1=12.a_1.\frac{H}{f}$

$\therefore\ \frac{W_1}{12.a_1.\frac{H}{f}}=$.283-lb. or .277-lb., according as the cable is made of steel or iron.

Let the safe inch-stress of steel be taken at 33,960-lbs., of the best cable-iron at 14,958-lbs., and of the best chain-links at 9,972-lbs.

$$\therefore W_1=H.\,a_1.\times.283\times\frac{12}{33,900}=\frac{H.a_1}{10,000},\text{ for steel cables.} \tag{16}$$

$$W_1=H.\,a_1\times.\,277\times\frac{12}{14,958}=\frac{H.a_1}{4,500},\text{ for iron cables.} \tag{17}$$

$$W_1=H.\,a_1\times.\,277\times\frac{12}{9,972}=\frac{H.a_1}{3,000},\text{ for link cables.} \tag{18}$$

Note.—About $\frac{1}{8}$th may be added to the net weight of a chain cable for eyes and fastenings.

(9).—*Deflection of a cable due to an elementary change in its length.*

By the Corollary of § (7), the total length (S) of the cable AOB is,

$$S=a_1+a_2+\frac{2}{3}.\frac{h_1^2}{a_1}.+\frac{2}{3}.\frac{h_2^2}{a_2}.$$

Now a_1 and a_2, are constant; h_1-h_2 is also constant, and $\therefore dh_1=dh_2$.

$$\text{Hence, } dS=\frac{4}{3}.\left(\frac{h_1}{a_1}+\frac{h_2}{a_2}\right).dh_1$$

If $h_1=h_2=h$, $\therefore\ a_1=a_2=\frac{a}{2}$,

and $dS=\frac{16}{3}.\frac{h}{a}.\,dh$ (19)

(10).—*Curve of cable; inclined suspenders.*—The condition of a cable from which the load is suspended by a series of sloping rods, is precisely the same as when the rods are vertical, except in the *direction* of the load.

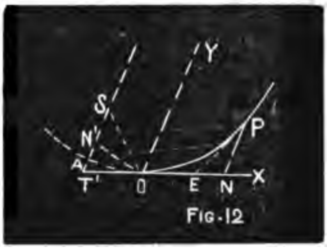


Fig. 12

Let O be the lowest point of such a cable. Let the tangent at O, and a line through O parallel to the suspenders, be the axes of x and y, respectively.

Let w' be the intensity of the oblique load. Consider a portion OP of the cable, and let the coordinates of P with respect to OX, OY, be x and y.

Draw the ordinate PN, and let the tangent at P meet ON in E.

As before, PNE is a triangle of forces, and E is the middle point of ON.

$$\therefore \frac{w'.x}{H}=\frac{PN}{NE}=\frac{2y}{x},\ \text{or}\ x^2=\frac{2H}{w'}\,y, \tag{20}$$

the equation to a parabola with its axis parallel to OY, and its focus at a point S where $4.\,SO=\frac{2.H}{w'}$.

Corollary 1.—Let the axis meet the tangent at O in T', and let its inclination to OX be i.

Let A be the vertex, and ON' a perpendicular to the axis.

$$\therefore SO=ST'=SA+AT'=SA+AN'$$

But, $4.AS.AN'=ON'^2=N'T'^2.\,tan^2 i=4.\,AN'^2.\,tan^2 i$

$\therefore AS=AN'.\,tan^2 i$, *and* $SO=AS.\,(1+Cot^2 i)=\frac{AS}{sin^2 i}$

Hence, the parameter$=4.\,AS=4.\,SO.\,sin^2 i.$ (21)

Corollary 2.—Let P be the oblique load upon the cable between O & P
" Q " " total thrust upon the platform at E.
" w " " load per horizontal unit of length.
" q " " rate of increase of thrust along platform.
" t " " length of PE.

$$\therefore w'=\frac{w}{sin\,i},\ and\ q=w.\,cot\,i \tag{22}$$

$$H=\frac{w'}{2}.\frac{x^2}{y}=2w'.SO=2AS.\frac{w'}{sin^2 i}=2.\ AS.\frac{w}{sin^3 i} \qquad (23)$$

$$P=H.\frac{2.t}{x}=\frac{w'.t.x}{y} \qquad (24)$$

$$t^2=y^2+\frac{x^2}{4}+x.y.\ Cos\ i \qquad (25)$$

Corollary 3.—Let s be the length of OP, and let θ be the inclination of PE to OY.

$$\therefore s=AP-AO$$

$$=\frac{H.sin^2 i}{2.w'}\Big\{tan\ (90-\theta).sec\ (90-\theta)+log_e(tan\ \overline{90-\theta}+sec.\ \overline{90-\theta})$$

$$-tan\ (90-i).sec\ (90-i)-log_e\ (tan\ \overline{90-i}+sec\ \overline{90-i})\Big\}$$

$$=\frac{H.\ sin^2 i}{2.\ w'}.\left\{cot\ \theta.cosec\ \theta-cot\ i.cosec\ i+log_e\frac{cot\ \theta+cosec\ \theta}{cot\ i+cosec\ i}\right\} \qquad (26)$$

It may be easily shewn as in the Corollary of §(7) that approximately,

$$s=x+y.cos\ i+\frac{2}{3}.\frac{y^2.\ sin^2 i}{x+y.\ cos\ i}. \qquad (27)$$

(11).—*Suspension bridge loads.*—The heaviest distributed load to which a highway bridge may be subjected is that due to a dense crowd of people, and is fixed by modern French practice at 82-lbs. per sq. ft. Probably, however, it is unsafe to estimate the load at less than from 100 to 140-lbs. per sq. ft., while allowance has also to be made for the concentration upon a single wheel of as much as 36,000-lbs., and perhaps more.

A procession marching in step across a suspension bridge may strain it far more intensely than a dead load, and will set up a synchronous vibration which may prove absolutely dangerous.

The *factor of safety* for the dead load of a suspension bridge should not be less than 2½ or 3, and for the live load it is advisable to make it 6. With respect to this point it may be remarked that the efficiency of a cable does not depend so much upon its ultimate strength as upon its limit of elasticity, and so long as the latter is not exceeded the cable remains uninjured. For example, the *breaking weight* of one of the 15-inch cables of the East River Bridge is estimated to be 12,000 tons, its *limit of elasticity* being 8,118-tons, so that with 1½ only as a factor of safety, the stress would still fall below the elastic limit and have no injurious effect. The *continual* application of such a load would doubtless ultimately lead to the destruction of the bridge.

The dip of the cable of a suspension bridge usually varies from 1-15th to 1-12th of the span, and is rarely as much as much as 1-10th, except for small spans.

(12).—*Modifications of the simple suspension bridge.*—

The disadvantages connected with suspension bridges are very great. The position of the platform is restricted, massive anchorages and piers are generally required, and any change in the distribution of the load produces a sensible deformation in the structure. Owing to the want of rigidity, a considerable vertical and horizontal oscillatory motion may be caused, and many efforts have been made to modify the bridge in such a manner as to neutralize the tendency to oscillation.

(*a*). – The simplest improvement is that shewn in Fig. 13, where the point of the cable most liable to deformation is attached to the piers by short straight chains *AB*.

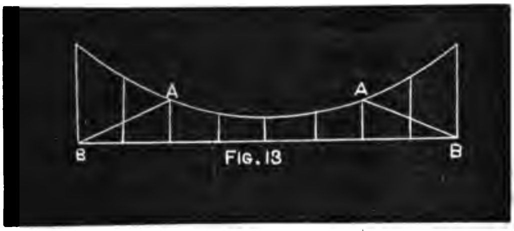


FIG. 13

(*b*). – A series of inclined stays, or iron ropes, radiating from the pier saddles, may be made to support the platform at a number of equidistant points, (Fig. 14.) Such ropes were used in the Niagara Bridge, and still more recently in the East River Bridge. The lower ends of the ropes are generally made fast to the top or bottom chord of the bridge truss, so that the corresponding chord stress is increased and the neutral axis proportionately displaced. To remedy this, it has been proposed to connect the ropes with a horizontal tie coincident in position with the neutral axis. Again, the cables of the Niagara and East River bridges do not hang in vertical planes, but are inclined inwards, the distance between them being greatest at the piers and least at the centre of the span. This *drawing in* adds greatly to the lateral stability, which may be still further increased by a series of horizontal ties.

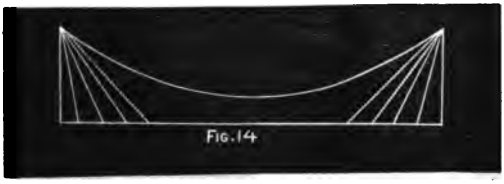


FIG. 14

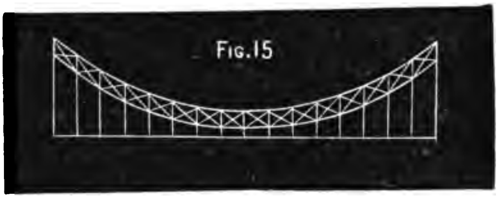


FIG.15

(*c*).–In Fig. 15 two cables in the same vertical plane are diagonally braced together. In principle this method is similar to that adopted in the ***stiffening truss***, (discussed in § 13), but is probably less efficient on account of the flexible character of the cables, although a slight economy of material might doubtless be realized. The braces act both as struts and ties, and the stresses to which they are subjected may be easily calculated.

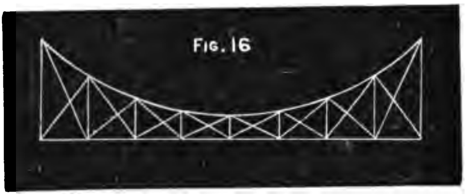


FIG. 16

(*d*).–In Fig. 16 a single chain is diagonally braced to the platform. The weight of the bridge must be sufficient to ensure that no suspender will be subjected to a thrust, or the efficiency of the arrangement is destroyed. An objection to this as well as to the preceding method is that the variation in the curvature of the chain under changes of temperature tends to loosen and strain the joints.

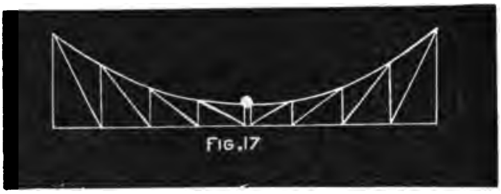


FIG.17

The principle has been adopted with greater prefection in the construction of a foot-bridge at Frankfort. The girder is cut at the centre, the chain is hinged, and the rigidity is obtained by means of vertical and inclined braces which act both as struts and ties (Fig. 17.)

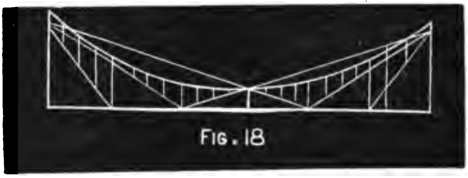


FIG. 18

(*e*).–In Fig. 18, the girder is supported at several points by straight chains running directly to the pier saddles, and the chains are kept in place by being hung from a curved chain by vertical rods.

(*f*).–It has been proposed to employ a stiff inverted arched rib of wrought-iron instead of the flexible cable. All straining action may be eliminated by hinging the rib at the centre and piers, and the theory of the stresses developed in this tension rib is precisely similar to that of the arched rib, except that the stresses are reversed in kind.

(*g*). – The platform of every suspension bridge should be braced horizontally. The floor beams are sometimes laid on the skew in order that the two ends of a beam might be suspended from points which do not oscillate concordantly, and also to distribute the load over a greater length of cable.

(13).—*Auxiliary or stiffening truss.* — The object of a stiffening truss (Fig. 19) is to distribute a passing load over the cable in such a manner that it cannot be distorted. The pull upon each suspender must therefore be the same, and this virtually assumes that the effect of the extensibility of the cable and suspenders upon the figure of the stiffening truss may be disregarded.

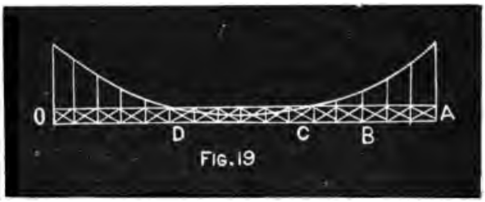


FIG. 19

The ends *O* and *A* must be anchored, or held down by pins, but should be free to move horizontally.

Let there be *n* suspenders, and let *T* be the pull upon each.

$\therefore$ t, the intensity of the pull per unit of span $= T \div \frac{l}{n+1} = \overline{n+1} \cdot \frac{T}{l}$, l being the span.

Let w be the uniform intensity of the dead load.

Let R_1, R_2, be the reactions at *O* and *A*, respectively.

CASE I.—*The bridge partially loaded.*

Let w' be the greatest uniform intensity of the live load, and let it advance from *A*, and cover a length *AB*.

Let $OB = x$

For equilibrium,

$$R_1 + R_2 + (t - w).\, l - w'.\,(l - x) = 0 \qquad (1)$$

$$\text{and, } R_1.\, l + \frac{t - w}{2}.\, l^2 - \frac{w'}{2}.\,(l - x)^2 = 0 \qquad (2)$$

Also. since the whole of the weight is to be transmitted through the suspenders, $(t - w).l - w'.(l - x) = 0$

$$\text{or, } t - w = \frac{w'}{l}.(l - x) \qquad (3)$$

From equations (1), (2), and (3),

$$-R_1=\frac{w'}{2}\cdot\frac{x}{l}.(l-x)=R_2 \tag{4}$$

which shews that the reactions at O and A are equal in magnitude but opposite in kind. They are evidently greatest when $x=\frac{l}{2}$, *i.e.*, when the live load covers half the bridge, and the common value is then $\frac{w'.l}{8}$.

The *shearing force* at any point *between O and B* distant x' from O,

$$=R_1+(t-w).x'=w'.\frac{l-x}{l}.\left(x'-\frac{x}{2}\right) \tag{5}$$

which becomes $\frac{w'}{2}.\frac{x}{l}.(l-x)=-R_1=R_2$, when x' equal x. Thus, the shear at the head of the live load is equal in magnitude to the reaction at each end, and is an absolute maximum when the live load covers half the bridge. The web of the truss must therefore be designed to bear a shear of $\frac{w'.l}{8}$ at the centre and ends.

Again, the *bending moment* at any point between O and B, distant x' from O,

$$=R_1\,x'+\frac{t-w}{2}.x'^2=\frac{w'}{2}.\frac{l-x}{l}.(x'^2-x.x'), \tag{6}$$

which is greatest when $x'=\frac{x}{2}$, *i.e.*, at the centre of OB, its value then being $-\frac{w'}{8}.\frac{l-x}{l}.x^2$. Thus, the bending moment is an *absolute maximum* when $\frac{d}{dx}(l.x^2-x^3)=0$, *i.e.*, when $x=\frac{2}{3}.l$, and its value is then $-\frac{w'}{54}.l^3$.

The bending moment at any point *between B and A* distant x' from O,

$$=R_1.x'+\frac{t-w}{2}.x'^2-\frac{w'}{2}.(x'-x)^2=\frac{w'}{2}.\frac{x}{l}.\overline{x'-x}.\overline{l-x'} \tag{7}$$

which is greatest when $\frac{d}{dx'}(\overline{x'-x}.\,\overline{l-x'})=0$, *i.e.*, when $x'=\frac{l+x}{2}$, or at the centre of AB, its value then being $\frac{w'}{8}.\frac{x}{l}.(l-x)^2$. Thus, the bending moment is an *absolute maximum* when $\frac{d}{dx}.(x.\overline{l-x}^2)=0$, *i.e.*, when $x=\frac{l}{3}$, and its value is then $+\frac{w'}{54}.l^3$.

Hence, *the maximum bending moments of the unloaded and loaded divisions of the truss are equal in magnitude but opposite in direction, and occur at the points of trisection (D, C) of OA, when the live load covers oners one-third (AC) and two-thirds (AD) of the bridge, respectively.*

Each chord must evidently be designed to resist both tension and compression, and in order to avoid unnecessary nicety of calculation, the section of the truss may be kept uniform throughout the middle half of its length.

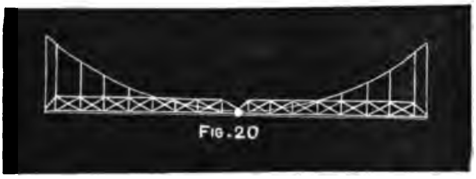


FIG. 20

In order that the truss might act most efficiently, it should be made in two halves, (Fig. 20), connected at the centre by a bolt strong enough to resist the shear of $\frac{w'.l}{8}$. The trusses can then deflect freely and are not sensibly strained by a rise or fall of the cable under a change of temperature.

Case II.—*A single concentrated load W at any point B of the truss.* *W* now takes the place of the live load of intensity w'.

The remainder of the notation and the method of procedure being precisely the same as before, the corresponding equations are,

$$R_1+R_2+(t-w).l-W=0 \qquad (1')$$

$$R_1.l+\frac{t-w}{2}.l^2-W(l-x)=0 \qquad (2')$$

$$t-w=\frac{W}{l} \qquad (3')$$

$$-R_1=\frac{W}{l}.\left(x-\frac{l}{2}\right)=R_2, \qquad (4')$$

which shews that the reactions at O and A are equal in magnitude but opposite in kind. They are greatest when $x=0$ and when $x=l$, *i.e.*, when W is either at O or at A, and the common value is then $\frac{W}{2}$

The *shearing force* at any point *between O and B* distant x' from O,

$$=R_1+(t-w).x'=\frac{W}{l}.\left(x'-x+\frac{l}{2}\right), \qquad (5')$$

which is a maximum when $x'=x$, and its value is then $\frac{W}{2}$.

The web must therefore be designed to bear a shear of $\frac{W}{2}$ throughout the whole length of the truss.

Again, the bending moment at any point *between O and B* distant x' from O,

$$=R_1.x' + (t—w).\frac{x'^2}{2} = \frac{W}{l}.\left\{\frac{x'^2}{2} + x'.\left(\frac{l}{2} - x\right)\right\} \qquad (6')$$

First let $x < \frac{l}{2}$. The bending moment is *positive* and is a maximum when $x' = x$, its value then being $+\frac{W}{2.l}(l.x - x^2)$.

Next let $x > \frac{l}{2}$. The bending moment is then *negative* and is a maximum when $x' = x - \frac{l}{2}$, its value then being $-\frac{W}{2.l}.\left(x - \frac{l}{2}\right)^2$.

The bending moment at any point *between B and A* distance x' from O

$$=R_1.x' + (t - w).\frac{x'^2}{2} - W.(x' - x) = \frac{W}{l}.(x' - l)\left(\frac{x'}{2} - x\right) \qquad (7')$$

which is a maximum when $\frac{d}{dx'}\left\{\overline{x' - l}.\left(\frac{x'}{2} - x\right)\right\} = 0$, *i.e.*, when

$$x' = x + \frac{2}{l}, \text{ and its value is then } -\frac{W}{2.l}.\left(x - \frac{l}{2}\right)^2.$$

Note.—The stiffening truss is most effective in its action, but adds considerably to the weight and cost of the whole structure. Provision has to be made both for the extra truss and for the extra material required in the cable to carry this extra load.

CHAPTER V.

ARCHED RIBS.

(1).—*Linear arches.*—When a cord hangs from two points of support, and is loaded with a number of weights, it tends to assume the form of a *funicular*, or *equilibrium* polygon, and the horizontal pull at every point of the cord is the same, (§ (3), Chap. I).

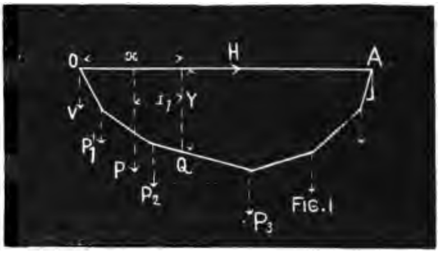


FIG. 1

Let O and A be the two points of support in the same horizontal plane.

Let x, y, be the co-ordinates of any point Q of the cord with respect to O.

Let P be the resultant of the weights between O and Q.

Let x_1 be the horizontal distance of P from Q.

Let V and H, respectively, be the vertical and horizontal components of the resultant force at O.

Take moments about Q.

$$H . y = V . x + P . x_1 \tag{1}$$

Suppose the cord to be exactly *inverted* and to be stayed in such a manner that it retains its form unchanged. Also, let the weights be the same in magnitude and distribution. The stresses at different points of the cord are now *reversed* in *kind* but *not* in *magnitude*, and equation (1) becomes,

$$H . y = V . x - P . x_1 = M \tag{2}$$

M being the bending moment at a distance x from O in a horizontal girder of the same span and similarly loaded.

But H is constant and $\therefore\ y \propto M$.

Hence, *the vertical ordinates at points of the inverted cord measured from the horizontal axis O A are proportional to the bending moments for corresponding sections of a girder of the same span and similarly loaded.*

If the number of the weights is increased indefinitely, and the distance between the points of application indefinitely diminished, the inverted cord becomes a curve, and forms a *linear arch*, or *equilibrated rib*.

An infinite number of linear arches, or *curves of equilibrium*, may be drawn through the points of support, as they are merely the bending moment curve plotted to different scales. These arches transmit a thrust only, and can never be constructed in practice, as the equilibrium is destroyed by the slightest variation in the distribution of the load from that for which the arch is designed, but the principles upon which they depend may be applied in the determination of the *lines of resistance* for arches and arched ribs.

Corollary 1.—If H be unity, the ordinates (y) are equal to the bending moments.

Corollary 2.—The linear arch for a load uniformly distributed along the horizontal is a *parabola*, (§ (4), Chap. IV).

(2).—*Arched Ribs.*—An arched rib is any truss in which both the chords are concave or convex in a vertical plane, and which gives rise to oblique reactions at the supports.

Arches and suspension bridges are the two extremes in which the horizontal stresses developed by the load are met by horizontal forces applied at the points of support, instead of being made to neutralize each other within the structure itself, and are therefore the only systems in which the material should or might be subjected to the *same kind* of stress throughout.

Wrought-iron and mild steel are very suitable for small arches in which parts of the rib are liable to tensile tresses. Cast-iron and cast-steel, having a greater compressive strength, might seem better adapted to large spans, as they admit of greater economy, but they are ill-suited to withstand shocks, and have been almost entirely superseded by the more elastic materials.

The rib may consist of a number of tubes bolted together, as in the St. Louis steel bridge, but is usually of an *I*-section. The depth of the rib is not necessarily uniform from end to end, but may be advantageously increased in parts subjected to an excessive bending action.

The space between the roadway and rib, *i.e.*, the spandril, may be filled up with some kind of lattice work, which should be very carefully designed, as the artistic appearance of the arch is largely dependent thereon.

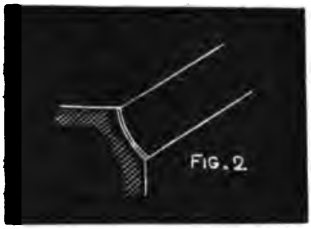


FIG. 2

Generally, the ends of the rib are securely bolted to iron skewbacks built into the abutments. It is far preferable to support the ends on pins or on cylindrical bearings, Fig. 2, which allow of a free rotational movement at the springings, so that the straining action in those parts due to changes of temperature or other causes are eliminated. Besides, the *horizontal* thrust, which is indeterminate when the ends are fixed, now becomes approximately axial and determinate, and a consideration of the actual deformation of different portions of the rib will render its strength completely ascertainable. Similar results have been obtained in roofs by suspending the ribs from single links attached to the supports.

The bending action at any point of the rib may be made to vanish by the introduction of a hinge, and the linear arch must pass through all such points. With a live load, however, there cannot be more than *three* hinges, or the equilibrium will be unstable and the arch will fail.

The practice of constructing timber arches with bent planks, bolted and scarfed together, is not to be recommended. When wood is employed, the arch should be a frame of which the several members are straight balks.

Timber arches are usually in the form of a segment of a circle, and a semi-circle is often employed for the sake of architectural appearance.

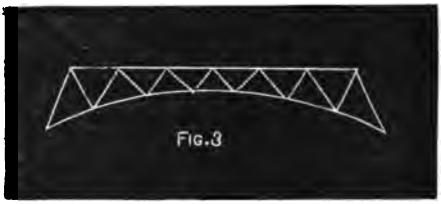


FIG.3

Fig. 3 represents a good form of metal arch for moderate spans, the chords being braced together, so as to prevent distortion. If the span is large, the depth of the rib may be increased and kept uniform throughout. The chords are then parallel, and the necessity for long braces at the abutments is obviated.

A horizontal distributing girder introduced at the top of an arched rib gives an increased depth, and is also intended to share the variable stresses induced by a passing load, so that the maximum stress at any point of the rib may never exceed the greatest stress produced when the bridge is completely covered by the passing load. Much uncertainty, however, exists as to the mutual relation of the girder and rib, and it seems better to concentrate the additional metal in the rib.

With a passing load the thrusts upon a pier supporting the ends of adjacent arches in a viaduct are often unequal, and if the conditions of stability will allow the pier to bear only a part of the resultant thrust, the remainder must be converted into an artificial thrust along the viaduct.

Note. – When the ends of a rib are hinged, H is indeterminate within limits dependent upon the frictional bending moment at the bearings.

(3).—*General proposition.*—It is now proposed to describe the method of calculating the *probable* maximum stresses at the different sections of an arched rib. The stresses cannot be found with mathematical accuracy, as they depend upon a variety of conditions which are altogether indeterminate.

The total stress at any point is made up of a number of subsidiary stresses, of which the most important are: (1) a direct thrust, (2) a stress due to flexure, (3) a stress due to a change of temperature. Each of these may be investigated separately.

(4).—*Bending moment* (M) *at any point of an arched rib.*

Let A B C, the axis of an arched rib *hinged* at the ends, coincide with a linear arch for a given load. The thrust at any cross-section of the rib is necessarily axial and uniformly distributed.

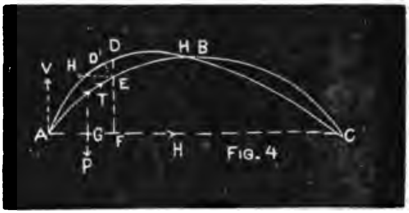


FIG. 4

For a different load the linear arch assumes a new form, as A D C, and the stress is no longer axial.

Let V and H, respectively, be the vertical and horizontal components of the thrust at A.

Let P, the resultant of the load between A and any ordinate D E F, meet A C in G.

$$\therefore M \text{ at } E, = V.\,\mathrm{AF} - P.\,\mathrm{GF} - H.\,\mathrm{EF}$$

$$\text{But, } 0 = V.\,\mathrm{AF} - P.\,\mathrm{GF} - H.\,\mathrm{DF}$$

$$\therefore M = H.(\mathrm{DF} - \mathrm{EF}) = H.\,\mathrm{DE} \qquad (1).$$

Hence, *the bending moment at any point of the axis is proportional to the vertical distance of that point from the linear arch.*

The same is true if the ends of the rib are absolutely *fixed.*

Let M_1 be the moment due to the fixture at A.

$$\therefore M = V.\,\mathrm{AF} - P.\,\mathrm{GF} - H.\,\mathrm{EF} - M_1$$

$$\& \ 0 = V.\,\mathrm{AF} - P.\,\mathrm{GF} - H.\,\mathrm{DF} - M_1$$

$$\& \ M = H.\,\mathrm{DE}, \text{ as before.} \qquad (2).$$

M is zero at H, the point of intersection of the axis and linear arch, and is negative, *i.e.*, changes in direction, at points on the right of H.

Corollary.—Let T be the thrust along the axis at E; draw D′E a normal at E.

$$\therefore T = H.\sec \text{DED}' = H.\frac{\text{DE}}{\text{D}'\text{E}}$$

$$\text{and } \because M = H.\,\text{DE} = T.\,\text{D}'\text{E}$$

(5). *Rib with hinged ends; invariability of span.*

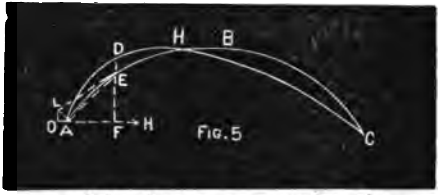


FIG. 5

Let ABC be the axis of a rib supported at the ends on pins or on cylindrical bearings. The resultant thrusts at A and C must necessarily pass through the centres of rotation. The vertical components of the thrusts are equal to the corresponding reactions at the ends of a girder of the same span and similarly loaded; H remains to be found.

Let ADC be a linear arch for any given distribution of the load, and let it intersect the axis of the rib at H. The curvature of the more heavily loaded portion AEH will be *flattened*, while that of the remainder will be sharpened.

The bending action at E tends to change the inclination of the rib at that point, and if the end A were free to move, the line EA would take the position EL, AL being of course very small; draw the vertical LO.

AO represents the horizontal displacement of the point E.

Since LAE is approximately a right-angle,

$$\therefore \text{AO} = \text{AL}.\frac{\text{EF}}{\text{AE}} \tag{1}$$

But the angle AEL = the change of curvature $= \dfrac{M}{E.I}$ nearly.

I being the moment of inertia of the section of the rib at E.

$$\therefore \text{AL} = \frac{M}{E.I}\text{AE, nearly,}$$

$$\text{and } \therefore \text{AO} = \frac{M}{E.I}.\text{EF} = H.\frac{\text{DE}.\text{EF}}{E.I} \tag{2}$$

But the length AC is assumed to be invariable, so that the total displacement between A and C is *nil*.

$$\therefore \Sigma\left(\frac{H.\text{DE}.\text{EF}}{E.\,I}\right) = 0$$

or, since H and E are constant,

$$\Sigma\left(\frac{\text{DE.EF}}{I}\right)=0 \qquad (3)$$

The *actual* linear arch may now be ascertained by drawing several linear arches and selecting the one which most nearly satisfies equation (3).

Corollary 1.—Equation (3) may be written,

$$\Sigma\left(\frac{\text{DF}-\text{EF}}{I}\right).\text{EF}=0, \text{ or } \Sigma\left(\frac{\text{DF.EF}}{I}\right)-\Sigma\left(\frac{\text{EF}^2}{I}\right)=0 \qquad (4)$$

Hence, the ordinates of the required arch may be directly calculated from the bending moments at the several sections of a girder of equal span and similarly loaded.

Corollary 2.—If the section of the rib is uniform, I is constant, and equations (3) and (4) respectively become,

$$\Sigma.(\text{DE.EF})=0 \text{ and } \Sigma(\text{DF.EF})-\Sigma(\text{EF}^2)=0$$

Corollary 3.—Let the axis of a rib of uniform section of span l and rise k, be a parabola, and let a single weight be placed upon the rib at a point of which the horizontal distance from the centre of the span is x.

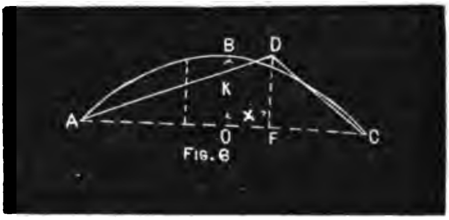


Fig. 6

The linear arch now consists of the two straight lines DA, DC, and it is found, by applying the condition (4), that $\text{D F}=\frac{32}{5}\cdot\frac{k.\,l^2}{5.l^2-4.x^2}$

Corollary 4.—If the axis of the rib is semi-circular, instead of a parabola, it is found that DF is equal to $\frac{M.l}{4}$, *i.e.*, is half the length of the rib.

No linear arch, however, will even approximately coincide with the axis of a rib rising vertically at the springings, so that a semi-circular or semi-elliptical axis is not to be recommended.

(6).—*Value of H.*—Let ADC be the *actual* linear arch of a rib of which the axis is ASC.

Let DF, the *maximum* ordinate of the arch, $=y$

Let x_1 be the horizontal distance from DF of P, the resultant of the load between A and DF.

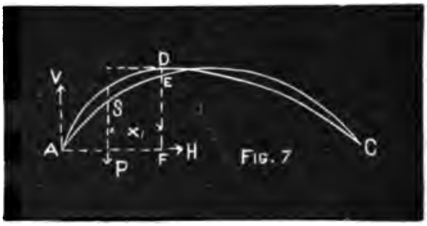


Fig. 7

Let AF$=x$

Take moments about D,

$$\therefore\ V.x-P.x_1=H.y$$

V and V', the vertical components of the thrusts at A and C, are equal to the corresponding reactions at the ends of a girder of the same span and similarly loaded. The resultant thrusts at A and C are therefore $\sqrt{V^2+H^2}$ and $\sqrt{V'^2+H^2}$, respectively.

The resultant thrust at any other point of the rib is now easily computed, and is resolvable into two components, the one normal to the section of the rib at the given point and the other in the plane of the section. The latter gives rise to a shearing stress and the former (unless axial) to a uniformly varying stress.

(7).—*Process of designing a rib.*—For a fixed distribution of the load the axis of the rib should be made to coincide with a linear arch for that load.

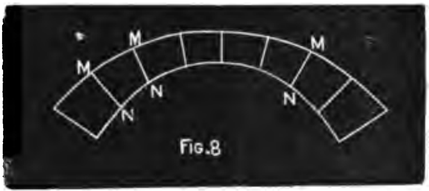


FIG. 8

Generally, the rib has to be designed to carry a *live*, in addition to a *dead* load, so that provision must be made for the stresses which may arise from every possible distribution of the former.

Let MN be a number of equi-distant cross-sections of a given rib. Draw a linear arch for each of the following cases:—

(1).–When the live load covers the *whole* of the bridge.

(2).–When the live load covers the *middle half* of the bridge.

(3).–When the live load covers *one-half* of the bridge from one end.

(4).–When the live load covers *three-fourths* of the bridge from one end.

Calculate the flange stresses in the different sections from the thrust in the nearest of the four linear arches, and add or remove metal according as the stress is excessive or falls below the safe limit. If the form of the rib is much changed by this process, a second approximation must be made.

(8).—*Rib with ends absolutely fixed.*—Let ABC be the axis of the rib. The fixture of the ends introduces two unknown moments, and since H is also unknown, three conditions must be satisfied before the strength of the rib can be determined.

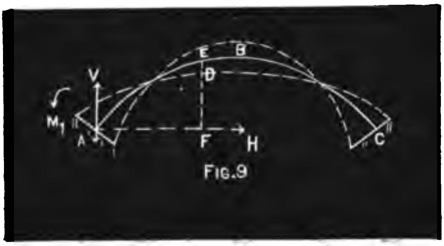


FIG. 9

The linear arch, as shewn by the dotted lines, may fall above or below the points A and C; draw any ordinate DF.

Since the ends of the rib are absolutely fixed, the total change of inclination of the rib between A and C must be zero.

But the change of inclination at any point E is $\frac{M}{E.I}$, approximately.

$$\therefore \Sigma\left(\frac{M}{E.I}\right)=0=\Sigma\left(\frac{H.\mathrm{DE}}{E.I}\right)$$

$$\text{or, } \Sigma.\left(\frac{\mathrm{DE}}{I}\right)=0 \qquad (1)$$

If the abutments are immovable the span AC is invariable; if they yield, the total horizontal displacement between A and C may be found by experiment; let it$=\mu.H$.

$$\therefore \text{by§5,} \Sigma\left(\frac{\mathrm{DE.EF}}{I}\right)=0,\ or=\mu.H \qquad (2)$$

The total *vertical* displacement between A and C must also be zero Referring to § 5 the vertical displacement of any point E is LO.

$$\text{But } \mathrm{LO}=\mathrm{AL}.\frac{\mathrm{AF}}{\mathrm{AE}}=\frac{M.\mathrm{AF}}{E.I}=\frac{H.\mathrm{DE.AF}}{E.I}$$

$$\therefore \Sigma\left(\frac{\mathrm{DE.AF}}{I}\right)=0 \qquad (3)$$

Equations (1), (2) and (3), are the three equations of condition.

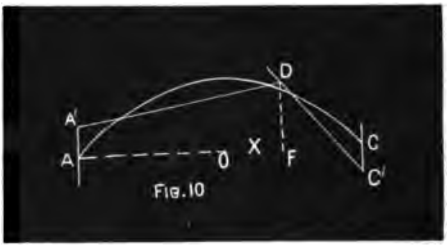


FIG. 10

Corollary 1.—Let the axis of the rib be a parabola of span l and rise k, and let the section of the rib be uniform.

Let a single weight be placed upon the rib at a point of which the horizontal distance from the centre of the span is x. The linear arch becomes the two straight lines DA′, DC′.

Let $\mathrm{AA}'=y_1, \mathrm{CC}'=y_2$, *and* $\mathrm{DF}=y$.

Condition (1) gives,

$$y.l+\left(\frac{l}{2}+x\right).y_1+\left(\frac{l}{2}-x\right).y_2=\frac{4}{3}.l.k$$

Condition (2) gives,

$$y.\left(\frac{5}{4}.l^2-x^2\right).l+\left(\frac{l}{2}+x\right)^2.\left(\frac{3}{2}.l-x\right).y_1+\left(\frac{l}{2}-x\right)^2.\left(\frac{3}{2}.l+x\right).y_2=\frac{8}{5}.k.l^3$$

Condition (3) gives,

$$y.\left(\frac{3}{2}.l-x\right).l+\left(\frac{l}{2}+x\right)\left(\frac{5}{2}.l-x\right).y_1+\left(\frac{l}{2}-x\right)^2.y_2=2.l^2.k$$

Solving these three equations,

$$\therefore y=\frac{6}{5}.k,\ y_1=\frac{2}{15}\cdot\frac{l+10.x}{l+2.x}\cdot k,\ \textit{and}\ y_2=\frac{2}{15}\cdot\frac{l-10.x}{l-2.x}.k.$$

(9).—*Effect of a change of temperature.*—The variation in the span l of an arch for a change of t° from the mean temperature is $\pm\, a.t.l$, approximately, a being the co-efficient of expansion.

Hence, if H_t is the horizontal force (thrust or tension) induced by the change of temperature, the condition that the length A C is invariable is expressed by the equation,

$$H_t.\Sigma\left(\frac{\text{DE.EF}}{E.I}\right)\pm a.t.l=0 \qquad (4)$$

Corollary 1.—Let a rib of uniform section of span l and rise k be hinged at the ends, Fig. 6. The linear arch is the line A C, and equation (4) becomes,

$$\frac{H_t}{E.I}.\Sigma(\text{EF}^2)\mp a.t.l=0=\frac{H_t}{E.I}.\frac{8}{15}.k^2.l\pm a.t.l$$

$$\therefore H_t=\mp\frac{15}{8}\cdot E.I.\frac{a.t}{k^2}.$$

The greatest bending action is at the crown, and is equal to $H_t.k$

Corollary 2.—If the axis of the rib in the previous Corollary is a semi-circle, instead of a parabola,

$$\therefore \frac{H_t}{E.I}.\Sigma(\text{EF}^2)\pm a.t.l=0=\frac{H_t}{E.I}.\frac{\pi}{16}.l^3\pm a.t.l$$

$$\therefore H_t=\mp\frac{16}{\pi}E.I.\frac{a.t}{l^2}$$

The greatest bending action is at the crown and is equal to $H_t.\frac{l}{2}$

Corollary 3.—Let the ends of the rib in Corollary 1 be absolutely fixed.

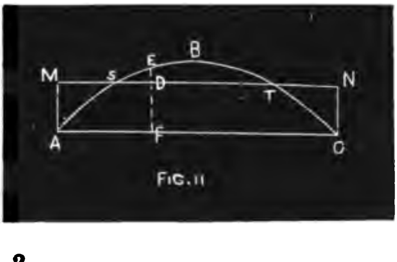


FIG. 11

The linear arch is a straight line MN which must satisfy condition 1, in § 8.

∴ the areas AMS and CNT are together equal to the area SBT,

$$\text{and}\ \therefore \text{MA}=\text{NC}=\frac{2}{3}.k.$$

Hence, equation (4) becomes,

$$\frac{H_t}{E.I}.\Sigma\left(\mathrm{EF}^2-\frac{2}{3}.k.\mathrm{EF}\right)\pm a.t.l=0=\frac{H_t}{E.I}.\left(\frac{8}{15}.k^2.l-\frac{4}{9}.k.l^2\right)\pm a.t.l$$

$$\therefore H_t=\mp\frac{45}{4}.E.I.\frac{a.t.}{k^2}.$$

The maximum bending action $=H_t.\frac{2}{3}.k=\mp\frac{15}{2}.E.I.\frac{a.t}{k}$

Corollary 4.—The co-efficient of expansion per degree of Fahrenheit is .0000062 and .0000067 for cast and wrought-iron beams, respectively. Hence, the corresponding total expansion or contraction in a length of 100 ft., for a range of 60° F. from the mean temperature is .0372 ft. $\left(=\frac{9''}{20}\right)$ and .0402 ft. $\left(=\frac{1''}{2}\right)$.

In practice the actual variation of length rarely exceeds *one-half* of these amounts, which is chiefly owing to structural constraint.

(10). — *Deflection of an arched rib.* — Let the abutments be immovable.

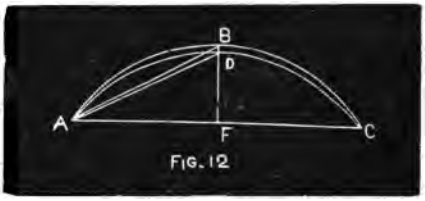


FIG. 12

Let ABC be the axis of the rib in its normal position.

Let ADC represent the position of the axis when the rib is loaded.

Let BDF be the ordinate at the centre of the span; join AB, AD.

$$\therefore \mathrm{DF}^2=\mathrm{AD}^2-\mathrm{AF}^2=\mathrm{AB}^2.\left(\frac{arc.\mathrm{AD}}{arc.\mathrm{AB}}\right)^2-\mathrm{AF}^2$$

But, $\dfrac{arc.\mathrm{AB}-arc.\mathrm{AD}}{arc.\mathrm{AB}}=\dfrac{f}{E}$,

f being the intensity of stress due to the change in the length of the axis.

$$\therefore \mathrm{DF}^2=\mathrm{AB}^2.\left(1-\frac{f}{E}\right)^2-\mathrm{AF}^2=\mathrm{BF}^2-\mathrm{AB}^2.\left\{2.\frac{f}{E}-\left(\frac{f}{E}\right)^2\right\}$$

$$\therefore \mathrm{AB}^2.\left\{2.\frac{f}{E}-\left(\frac{f}{E}\right)^2\right\}=\mathrm{BF}^2-\mathrm{DF}^2=(\mathrm{BF}-\mathrm{DF})\,(\mathrm{BF}+\mathrm{DF})$$

$$=2.\mathrm{BF}.(\mathrm{BD}).,\ \text{approximately.}$$

$\left(\frac{f}{E}\right)^2$ is also sufficiently small to be disregarded,

$$\therefore \mathrm{BD},\ \text{the deflection},=\frac{\mathrm{AB}^2}{\mathrm{BF}}.\frac{f}{E}=\frac{k^2+\frac{l^2}{4}}{k}.\frac{f}{E}$$

(11).—*Elementary deformation of an arched rib.*—

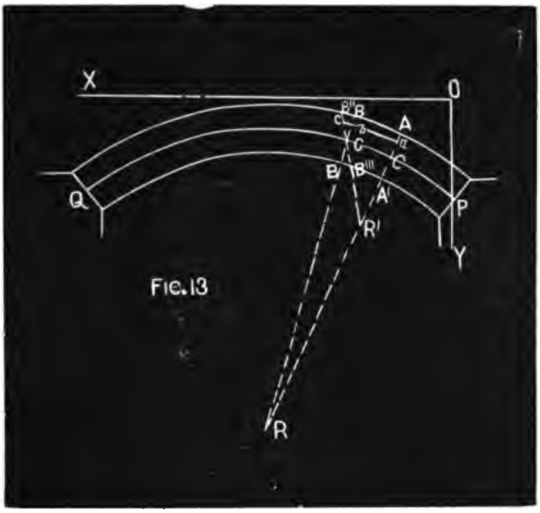


FIG. 13

The arched rib, represented by Fig. 13, springs from two abutments and is under a vertical load. The neutral axis PQ is the locus of the centres of gravity of all the cross-sections of the rib, and may be regarded as a linear arch, to which the conditions governing the equilibrium of the rib are equally applicable.

Let AA' be any cross-section of the rib. The segment $AA'P$ is kept in equilibrium by the external forces which act upon it, and by the molecular action at AA'.

The external forces are reducible to a single force at C and to a couple of which the moment M is the algebraic sum of the moments with respect to C of all the forces on the right of C.

The single force at C may be resolved into a component T along the neutral axis, and a component S in the plane AA'. The latter has very little effect upon the curvature of the neutral axis, and may be disregarded as compared with M.

Before deformation let the consecutive cross-sections BB' and AA' meet in R; R is the centre of curvature of the arc CC' of the neutral axis.

After deformation it may be assumed that the plane AA' remains unchanged, but that the plane BB' takes the position $B''B'''$. Let AA' and $B''B'''$ meet in R'; R' is the centre of curvature of the arc CC' *after* deformation.

Let abc be any layer at a distance z from C.

Let $CC'' = \Delta s$, $CR = R$, $CR' = R'$, and let Δa be the sectional area of the layer abc.

By similar figures, $\dfrac{ac}{\Delta s} = \dfrac{R' + z}{R'}$, and $\dfrac{ab}{\Delta s} = \dfrac{R + z}{R}$.

$$\therefore\ bc = ac - ab = \Delta s.z.\left(\frac{1}{R'} - \frac{1}{R}\right)$$

The tensile stress in $abc = E.\Delta a.\dfrac{bc}{ab} = E.\Delta a.\dfrac{\Delta s.z}{ab}\cdot\left(\dfrac{1}{R'} - \dfrac{1}{R}\right)$

$$= E.\Delta a\cdot z.\left(\frac{1}{R'} - \frac{1}{R}\right), \text{ very nearly.}$$

The moment of this stress with respect to $C = E.\Delta a.z^2.\left(\dfrac{1}{R'} - \dfrac{1}{R}\right)$

Hence, the moment of resistance at AA'

$$= \int E.\Delta a.z^2.\left(\frac{1}{R'} - \frac{1}{R}\right) = E.\left(\frac{1}{R'} - \frac{1}{R}\right)\cdot\int \Delta a.z^2,$$

the integral extending over the whole of the section.

$$\therefore M = E.I.\left(\frac{1}{R'} - \frac{1}{R}\right) \qquad (1)$$

Again, the effect of the force T is to lengthen or shorten the element CC'', so that the plane BB' will receive a motion of translation, but the position of R' is practically unaltered.

Corollary 1.—Let A be the area of the section AA'.

The total unit stress in the layer $abc = p = \dfrac{T}{A} \pm \dfrac{M.z}{I}$, $\qquad$ (2)

the sign being *plus* or *minus* according as M acts towards or from the edge of the rib under consideration.

From this expression may be deduced, (1).—the position of the point at which the intensity of the stress is a maximum for any given distribution of the load, (2).—the distribution of the load that makes the tensity an absolute maximum, (3).—the value of the intensity.

Corollary 2.—Let w be the total intensity of the vertical load per horizontal unit of length.

Let w_1 be the portion of w which produces only a direct compression.

Let H be the horizontal thrust of the arch.

Let P be the total load between the crown and AA' which produces compression.

Refer the rib to the horizontal OX and the vertical OPY as the axes of x and y, respectively.

Let x, y, be the co-ordinates of C.

$$\therefore P=H.\frac{dy}{dx}\text{; but } dP=w_1.dx$$

$$\therefore\ w_1=H.\frac{d^2y}{dx^2} \tag{3}$$

$$\text{also } T=H.\frac{ds}{dx} \tag{4}$$

(12).—*General equations.*

Let x, y, be the co-ordinates of the point C *before* deformation.
" x',y', " " " " *after* "
" θ be angle between tangent at C and OX *before* deformation.
" $\theta'(=\theta-\Delta\theta)$ " " " *after* "
" ds be the length of the element CC'' before deformation.
" ds' " " " after "

Effect of flexure. $\dfrac{d\theta'}{ds'}=\dfrac{1}{R'}$, and $\dfrac{d\theta}{ds}=\dfrac{1}{R}$

$$\therefore \frac{M}{E.I}=\frac{1}{R'}-\frac{1}{R}=\frac{d\theta'}{ds'}-\frac{d\theta}{ds}=\frac{d\theta'-d\theta}{ds}\text{, very nearly.}$$

$$\therefore \Delta\theta=\theta-\theta'=\Delta\theta_0-\int_0^x\frac{M}{E.I}.\frac{ds}{dx}.dx, \tag{5}$$

$\Delta\theta_0$ being the *alteration of slope* at P, which has yet to be determined.

Again, $\cos\theta'=\cos(\theta-\Delta\theta)=\cos\theta+\Delta\theta.\sin\theta$
and $\sin\theta'=\sin(\theta-\Delta\theta)=\sin\theta-\Delta\theta.\cos\theta$

$$\therefore \frac{dx'}{ds'}=\frac{dx}{ds}+\Delta\theta.\frac{dy}{ds}\text{, and }\frac{dy'}{ds'}=\frac{dy}{ds}-\Delta\theta.\frac{dx}{ds}. \tag{6}$$

Effect of T and of a change of t^o in the temperature.—
ds is compressed by T, and $\therefore$ its new length due to the compression $=ds.\left(1-\dfrac{T}{E.A}\right)$

In order to make allowance for the change in temperature, this last expression must be multiplied by $(1\pm a.t)$, a being the co-efficient of linear dilatation of the material of the rib.

$$\therefore ds'=ds.\left(1-\frac{T}{E.A}\right)\cdot(1\pm a.t). \tag{7}$$

Hence, by equations (6),

$$dx'=(dx+\Delta\theta.dy).\left(1-\frac{T}{E.A}\right).(1\pm a.t)$$

$$=dx+\Delta\theta.dy-\frac{T}{E.A}\cdot dx\pm a.t.dx\text{, approximately.}$$

and $dy'=(dy-\Delta\,\theta.dx).\left(1-\frac{T}{E.A}\right)\,(1\pm a.t)$

$=dy-\Delta\theta.dx-\frac{T}{E.A}.dy\pm a.t.dy,$ approximately

or, $dx'-dx=\Delta\,\theta.\frac{dy}{dx}.dx-\frac{T}{E.A}.dx\pm a.t.dx$

and $dy'-dy=-\Delta\,\theta.dx-\frac{T}{E.A}.\frac{dy}{dx}.dx\pm a.t.\frac{dy}{dx}.dx$

$$\therefore\ x'-x=\int_0^x\Delta\,\theta.\frac{dy}{dx}.dx-\int_0^x\frac{T}{E.A}.dx\pm\int_0^x a.t.dx \qquad (8)$$

$$\text{and } y'-y=-\int_0^x\Delta\,\theta.dx-\int_0^x\frac{T}{E.A}.\frac{dy}{dx}.dx\pm\int_0^x a.\,t.\,\frac{dy}{dx}.dx \qquad (9)$$

Let l be the span of the arch.

Let $x_0(=0), y_o$, be the values of x, y, at P.

Let $\Delta\,\theta_1$, x_1, y_1, be the values of $\Delta\,\theta$, x', y', respectively, at Q.

$$\therefore\Delta\,\theta_1=\Delta\,\theta_o-\int_0^l\frac{M}{E.I}.\frac{ds}{dx}.dx \qquad (10)$$

$$x_1-l=\int_0^l\Delta\,\theta.\frac{dy}{dx}.dx-\int_0^l\frac{T}{E.A}.dx\pm\int_0^l a.t.dx. \qquad (11)$$

$$y_1-y_o=\int_0^l\Delta\,\theta.dx-\int_0^l\frac{T}{E.A}.\frac{dy}{dx}.dx\pm\int_0^l a.t.\frac{dy}{dx}dx \qquad (12)$$

Again, the general equations of equilibrium at the plane AA' are,

$$\frac{d^2M}{dx^2}=\frac{dS}{dx}=-(w-w_1)=-\left(w-H.\frac{d^2y}{dx^2}\right), \qquad (13)$$

for the portion w_1, Cor.3,§(11), produces compression only and no shear,

$$\therefore\ S=S_o-\int_0^x w.dx+H.\left(\frac{dy}{dx}-\frac{dy_o}{dx_o}\right) \qquad (14)$$

S_o being the still undetermined vertical component of the shear at P, and $\frac{dy_o}{dx_o}$ the slope at P,

$$\text{and, } M=M_o+S_o.x-\int_0^x\int_0^x w.dx^2+H.\left(y-y_o-x.\frac{dy_o}{dx_o}\right) \qquad (15)$$

M_o being the still undetermined bending moment at P.

Equations (8), (9), (14) and (15) contain the *four* undetermined constants $H, S_o, M_o, \Delta\theta_o$, T being $H.\frac{ds}{dx}$

Let M_1, S_1, be the values of M and S, respectively, at Q.

Equations of condition.—In practice the ends of the rib are either *fixed* or *free*.

If they are fixed $\Delta\,\theta_0=0$, if they are free $M_0=0$. In either case the number of undetermined constants reduces to *three*.

If the abutments are immovable, $x_1-l=0$. If the abutments yield, x_1-l must be found by experiment; let $x_1-l=\mu.H$, μ being some coefficient.

The *first* equation of condition is $x_1-l=0$, or, $x_1-l=\mu.H$ (16)

Again, Q is immovable in a vertical direction.

The *second* equation of condition is, $y_1-y_0=0$ (17)

Again, if the end Q is fixed, $\Delta\theta_1=0$, and if free, $M_1=0$

The *third* equation of condition is $\Delta\theta_1=0$, or $M_1=0$ (18)

Substituting in equations (14) and (15) the values of the three constants as determined by these conditions, the shearing force and bending moment may be found at any section of the rib.

(13).—*Rib of uniform stiffness.*—Let the *depth* and *sectional form* of the rib be uniform, and let its breadth at each point vary as the *secant* of the inclination of the tangent at the point to the horizontal.

Let A_1, I_1, be the sectional area and moment of inertia at the crown
" A, I, " " " at any point C.

$$\therefore\ A=A_1.\sec\theta=A_1.\frac{ds}{dx} \tag{19}$$

Also, since the moments of inertia of similar figures vary as the breadth and as the cube of the depth, and since the depth in the present case is constant,

$$\therefore I=I_1.\sec\theta=I_1.\frac{ds}{dx} \tag{20}$$

Again, $\frac{T}{A}=\frac{H.\sec\theta}{A_1.\sec\theta}=\frac{H}{A_1}$, and the intensity of the thrust is constant throughout.

Hence, equations (5), (8), and (9), respectively become,

$$\Delta\,\theta=\Delta\,\theta_0-\frac{1}{E.I_1}.\int_0^x M\,dx \tag{21}$$

$$x'-x=\int_0^x \Delta\,\theta.\frac{dy}{dx}\,dx-\frac{H}{EA_1}\cdot x\pm a.t.x \tag{22}$$

$$y'-y=-\int_0^x \Delta\theta.\,dx-\frac{H}{E.A_1}(y-y_0)\pm a.t.(y-y_0) \tag{23}$$

Equation (21) shews that the deflection at each point of the rib is the same as that at corresponding points of a straight horizontal beam of a uniform section equal to that of the rib at the crown, and acted upon by the same bending moments.

Ribs of uniform stiffness are not usual in practice, but the formulæ deduced in the present article may be applied without sensible error to flat segmental ribs of uniform section.

(13).—*Parabolic rib of uniform depth and stiffness, with rolling load; the ends fixed in direction; the abutments immovable.*

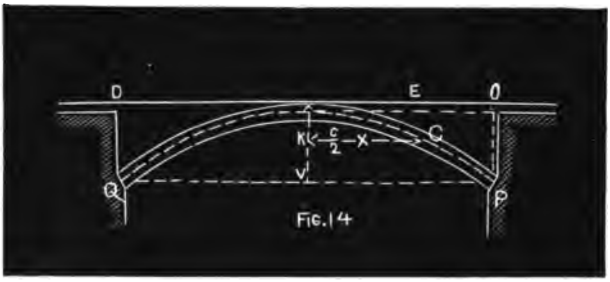


FIG. 14

Let the axis of x be a tangent to the neutral curve at its summit.

Let k be the rise of the curve.

Let x, y, be the co-ordinates at any point C with respect to O.

$$\therefore y=\frac{4.k}{l^2}\cdot\left(\frac{l}{2}-x\right)^2 \qquad (24)$$

$$\text{and, } \frac{dy}{dx}=-\frac{8k}{l^2}\cdot\left(\frac{l}{2}-x\right), \frac{dy_0}{dx_0}=-\frac{4.k}{l}\cdot\frac{dy_1}{dx_1}=\frac{4.k}{l}, \frac{d^2y}{dx^2}=\frac{8k}{l^2} \qquad (25)$$

Let w be the dead load per horizontal unit of length.

Let w' " live " " " " "

Let the live load cover a length $DE, = r.\, l$, of the span.

Denote by (A) formulæ relating to the unloaded division OE, and by (B) formulæ relating to the loaded division DE.

Equations (14) and (15) respectively become,

$$(A) \qquad S=S_0+\left(\frac{8.\,k.H}{l^2}-w\right).x \qquad (26)$$

$$(B) \qquad S=S_0+\left(\frac{8.\,k.\,H}{l^2}-w\right).x-w'.\{x-(1-r).l\} \qquad (27)$$

$$(A) \qquad M=M_0+S_0.x+\left(\frac{8.k.H}{l^2}-w\right)\cdot\frac{x^2}{2} \qquad (28)$$

$$(B) \qquad M=M_0+S_0.x+\left(\frac{8.k.H}{l^2}-w\right)\cdot\frac{x^2}{2}-\frac{w'}{2}\cdot\{x-(1-r).l\}^2 \qquad (29)$$

Since the ends are fixed, $\Delta\theta_o = 0 = \Delta\theta_1$ (30)

Hence, by equations (21) and (28),

$$(A) \qquad \Delta\theta = -\frac{1}{E.I_1}\cdot\left\{M_o.x + S_o.\frac{x^2}{2} + \left(\frac{8.k.H}{l^2} - w\right)\frac{x^3}{6}\right\} \tag{31}$$

and by equations (21) and (29),

$$(B) \qquad \Delta\theta = -\frac{1}{E.I_1}\cdot\left\{M_o.x + S_o.\frac{x^2}{2} + \left(\frac{8\,k.H}{l^2} - w\right)\cdot\frac{x^3}{6} - \frac{w'}{6}\cdot\{x - (1 - r).l\}^3\right\} \tag{32}$$

When $x=l$, $\Delta\theta = \Delta\theta_1 = 0$, and therefore by the last equation,

$$0 = M_o + S_o\cdot\frac{l}{2} + \left(\frac{8.k.H}{l^2} - w\right)\cdot\frac{l^2}{6} - \frac{w'}{6}.r^3.l^2 \tag{33}$$

Again, let $\Delta\theta = \dfrac{dv}{dx}$,

$$\therefore \int_o^l \Delta\theta.\frac{dy}{dx}.dx = \int_o^l \frac{dv}{dx}\cdot\frac{dy}{dx}.\,dx = \Delta\theta_1.\frac{dy}{dx} - \int_o^l v.\frac{d^2y}{dx^2}.dx$$

But $\Delta\theta_1 = 0$, and $\dfrac{d^2y}{dx^2} = \dfrac{8.k}{l^2}$,

$$\therefore \int_o^l \Delta\theta.\frac{dy}{dx}.dx = -\frac{8.k}{l^2}\cdot\int_o^l v.dx = -\frac{8.k}{l^2}\cdot\int_o^l\int_o^x \Delta\theta.dx^2 \tag{34}$$

By the conditions of the problem, $x' - x$ and $y' - y$ are each zero at Q. Hence, equations (22) and (23) respectively become,

$$0 = -\frac{8\,k}{l^2}\cdot\int_o^l\int_o^x \Delta\theta.dx^2 - \frac{H}{E.A_1}.l \pm a.t.l \tag{35}$$

$$0 = -\int^l \Delta\theta.dx \tag{36}$$

Substitute the value of $\Delta\theta$ in equation (32), and integrate between the limits;

$$\therefore 0 = -\frac{8.k}{l^2}\cdot\frac{1}{E.I_1}\cdot\left\{M_o\cdot\frac{l^3}{6} + S_o\cdot\frac{l^4}{24} + \left(\frac{8.k.H}{l^2} - w\right)\cdot\frac{l^5}{120} - w'.r^5.\frac{l^5}{120}\right\} - \frac{H}{E.A_1}.l \pm a.t.l.$$

$$\text{and, } 0 = -\frac{1}{E.I_1}\cdot\left\{\frac{M_o.l^2}{2} + \frac{S_o.l^3}{6} + \left(\frac{8.k.H}{l^2} - w\right)\cdot\frac{l^4}{24} - w'.r^4.\frac{l^4}{24}\right\}$$

which may be written,

$$0 = M_o + \frac{S_o}{4}\cdot l + \left(\frac{8\,k.H}{l^2} - w\right)\cdot\frac{l^2}{20} - \frac{w'}{20}.r^5.l^2 + \frac{3}{4}\cdot\frac{H.I_1}{A_1.k} \mp \frac{a.t}{8}\cdot\frac{E.I_1}{k} \tag{37}$$

$$\text{and, } 0 = M_o + \frac{S_o}{3}.l + \left(\frac{8\,k.H}{l^2} - w\right)\cdot\frac{l^2}{12} - w'.r^4.\frac{l^2}{12} \tag{38}$$

Equations (33), (37), and (38), are the equations of condition.

Subtract (37) from (33),

$$\therefore 0 = \frac{S_o}{4} \cdot l. + \left(\frac{8\,k.H}{l^2} - w\right) \cdot \frac{7}{60}.l^2 - w'.l^2.\left(\frac{r^3}{6} - \frac{r^5}{20}\right) - \frac{3}{4} \cdot \frac{H.I_1}{A_1.k} \pm \frac{a.t}{8} \cdot \frac{E.I_1}{k} \qquad (39)$$

Subtract (37) from (38), and multiply the result by 3,

$$\therefore 0 = \frac{S_o}{4} l. + \left(\frac{8\,k.H}{l^2} - w\right) \cdot \frac{l^2}{10} - w'.l^2.\left(\frac{r^4}{4} - \frac{3.r^5}{20}\right) - \frac{9}{4} \cdot \frac{H.I_1}{A_1.k} \pm \frac{3}{8}.a.t.\frac{E.I_1}{k} \qquad (40)$$

Subtract (39) from (40),

$$\therefore 0 = -\left(\frac{8.k.H}{l^2} - w\right) \cdot \frac{l^2}{60} - w'.l^2.\left(\frac{r^4}{4} - \frac{r^5}{10} - \frac{r^3}{6}\right) - \frac{3}{2} \cdot \frac{H.I_1}{A_1.k} \pm \frac{a.t.}{4} \cdot \frac{E.I_1}{k} \qquad (41)$$

$$\text{Hence, } H = \frac{l^2.\left\{\frac{w}{8} + w'\left(\frac{5}{4}.r^3 - \frac{15}{8}.r^4 + \frac{3}{4}.r^5\right)\right\} \pm \frac{15}{8}.a.t.\frac{E.I_1}{k}}{k.\left(1 + \frac{45}{4}.\frac{I_1}{a.k.^2}\right)} \qquad (42)$$

Eliminating S_o between (33) and (38),

$$\therefore M_o = \left(\frac{8.k.H}{l^2} - w\right) \cdot \frac{l^2}{12} - w'.l^2.\left(-\frac{r^5}{3} - \frac{r^4}{4}\right) \qquad (43)$$

By equation (29), M at Q is,

$$-M_1 = M_o + S_o.l. + \left(\frac{8.k.H}{l^2} - w\right) \cdot \frac{l^2}{2} - \frac{w'}{2}.r^2.l^2 \qquad (44)$$

Eliminating S_o between (33) and (44),

$$\therefore -M_1 = -M_o + \left(\frac{8.k.H}{l^2} - w\right) \cdot \frac{l^2}{6} - w'.l^2.\left(\frac{r^2}{2} - \frac{r^3}{3}\right) \qquad (45)$$

Substitute in this equation the value of M_o in (43),

$$\therefore -M_1 = \left(\frac{8.k.H}{l^2} - w\right) \cdot \frac{l^2}{12} - w'.\,l^2\left(\frac{r^2}{2} - \frac{2.r^3}{3} + \frac{r^4}{4}\right) \qquad (46)$$

To find the greatest intensity of stress, etc.—

The intensity of the stress due to direct compression $= \frac{T}{A} = \frac{H}{A_1}$

The intensity of the stress in the outside layers of the rib due to *bending* is the same as that in the outside layers of a horizontal beam of uniform section A_1 acted upon by the same moments as act on the rib, for the deflections of the beam and rib are equal at every point, (Equation 21). Also, since the rib is fixed at both ends, the bending moment due to that portion of the load which produces flexure is a *maximum* at the loaded end, *i.e.*, at Q. Hence, the maximum intensity of stress (p_1) occurs at Q, and $p_1 = \frac{H}{A_1} \pm M_1.\frac{z_1}{I_1}$ (47)

z_1 being the distance of the layers from the neutral axis.

H and M_1 are both functions of r, and therefore p_1 is an *absolute maximum* when $\frac{dp_1}{dr} = 0 = \frac{1}{A_1}.\frac{dH}{dr} \pm \frac{z_1}{I_1}.\frac{dM_1}{dr}$ (48)

But, by equation (42),

$$\frac{dH}{dr} = \frac{w'.l^2.\frac{15}{4}.r^2.(r-1)^2}{k.\left(1 + \frac{45}{4}.\frac{I_1}{A.k^2}\right)} \tag{49}$$

and by equation (46),

$$\frac{dM_1}{dr} = \frac{2}{3}.k.\frac{dH}{dr} - w'.l.^2 r.(r-1)^2 \tag{50}$$

$$\therefore 0 = \left(\frac{1}{A} \pm \frac{2}{3}.k.\frac{z}{I_1}\right).\frac{w'.l^2.\frac{15}{4}.r^2.(r-1)^2}{k.\left(1 + \frac{45}{4}.\frac{I_1}{A_1.k^2}\right)} \mp \frac{z_1}{I_1}.w'.l^2.r\,(r-1)^2 \tag{51}$$

which may be written,

$$0 = (r-1)^2.\left\{\frac{5}{2}.r.\left(\frac{3}{2A_1} \pm k.\frac{z_1}{I_1}\right) \mp \frac{z_1}{I_1}.k.\left(1 + \frac{45}{4}.\frac{I_1}{A_1.k^2}\right)\right\} \tag{52}$$

The roots of this equation are,

$$r = 1 \text{ and } r = \pm.\frac{2}{5}.\frac{1 + \frac{45}{4}.\frac{I_1}{A_1.k^2}}{\frac{3}{2}.\frac{I_1}{A_1.z_1.k} \pm 1} \tag{53}$$

$r = 1$ makes $\frac{d^2p}{dr^2}$ *zero*, so that the maximum value of p corresponds to one of the remaining roots.

Hence, the *maximum thrust* $= \frac{1}{A^1}.\left(H + \frac{A_1 z}{I_1}.M_1\right) = p'_1$ (54)

the values of H and M_1 being obtained by substituting in Equations (42) and (46),

$$r = \frac{2}{5}.\frac{1 + \frac{45}{4}.\frac{I_1}{A_1.k^2}}{1 - \frac{3}{2}.\frac{I_1}{A_1.z_1.k}}. \tag{55}$$

and the *maximum tension* $= \frac{1}{A_1}.\left(-H + \frac{A_1 z_1}{I} M_1\right) = p''_1$ (56)

the values of H and M_1 being obtained by substituting in Equations (42) and (46),

$$r = \frac{2}{5} \cdot \frac{1 + \frac{45}{4} \cdot \frac{I_1}{A_1 . k^2}}{1 + \frac{3}{2} \cdot \frac{I_1}{A_1 . z_1 . k}} \tag{57}$$

Equation (54) will give the proper sectional area when the depth and form of the rib have been fixed.

If Equation (56) gives a negative result there is no tension at any point of the rib.

Corollary 1.—The moment of inertia may be expressed in the form,

$$I_1 = q . z_1^2 . A_1,$$

q being a co-efficient which depends upon the figure of the section.

$$\therefore \text{ the } \textit{maximum} \text{ thrust } = \frac{1}{A_1} \cdot \left(H + \frac{M_1}{q . z_1} \right) \tag{58}$$

and the corresponding value of r is,

$$r = \frac{2}{5} \cdot \frac{1 + \frac{45}{4} . q . \frac{z_1^2}{k^2}}{1 - \frac{3}{2} \cdot q . \frac{z_1}{k}} = \frac{2}{5} \cdot \frac{b}{c}, \text{ suppose.} \tag{59}$$

$$\text{By (42), } H = \frac{\frac{w . l^2}{8} \pm \frac{15}{8} . a . t . \frac{E . I_1}{k} + w' . l^2 . \left(\frac{5}{4} . r^3 - \frac{15}{8} . r^4 + \frac{3}{4} . r^5 \right)}{k . b} \tag{60}$$

$$\text{By (46), } M_1 = - \frac{2}{3} . k . H + \frac{w . l^2}{12} + w' . l^2 . \left(\frac{r^2}{2} - \frac{2}{3} . r^3 + \frac{r^4}{4} \right)$$

$$= \frac{\frac{15}{16} . w . l^2 . q . \frac{z_1^2}{k^2} \mp \frac{5}{4} . a . t . \frac{E . I_1}{k} - w' . l^2 . \left(\frac{5}{6} . r^3 - \frac{5}{4} . r^4 + \frac{r^5}{2} \right)}{b} + w' . l^2 . \left(\frac{r^2}{2} - \frac{2}{3} . r^3 + \frac{r^4}{4} \right)$$

$$\therefore H_1 + \frac{M_1}{q . z_1} = \frac{w . l^2}{8 . b} \cdot \left(\frac{1}{k} + \frac{15}{2} \cdot \frac{z_1}{k^2} \right) \mp \frac{5}{4} \cdot \frac{a . t . E . I_1}{q . z_1 . k . b} \cdot \left(1 - \frac{3}{2} \cdot \frac{q . z_1}{k} \right)$$

$$+ \frac{w' . l^2}{q . z_1 . b} \cdot \left\{ \frac{q . z_1}{k} \cdot \left(\frac{5}{4} . r^3 - \frac{15}{8} . r^4 + \frac{3}{4} . r^5 \right) - \left(\frac{5}{6} . r^3 - \frac{5}{4} . r^4 + \frac{r^5}{2} \right) \right\} + \frac{w' . l^2}{q . z_1} \cdot \left(\frac{r^2}{2} - \frac{2}{3} . r^3 + \frac{r^4}{4} \right)$$

$$= \frac{w . l^2}{8 . b} \cdot \left(\frac{1}{k} + \frac{15}{2} \cdot \frac{z_1}{k^2} \right) \mp \frac{5}{4} \cdot \frac{a . t . E . I_1}{q . z_1 . k} \cdot \frac{c}{b} - \frac{w' . l^2}{q . z_1} \cdot \left\{ \frac{5}{6} . r^3 - \frac{5}{4} . r^4 + \frac{r^5}{2} \right\} \left\{ \frac{1 - \frac{3}{2} \cdot \frac{q . z_1}{k}}{b} \right\}$$

$$+ \frac{w' . l^2}{q . z_1} \cdot \left(\frac{r^2}{2} - \frac{2}{3} . r^3 + \frac{r^4}{4} \right)$$

But $1 - \frac{3}{2} \cdot \frac{q . z_1}{k} = c$ and $r = \frac{2}{5} \cdot \frac{b}{c}$

$$\therefore H_1 + \frac{M_1}{q . z_1} = \frac{w . l^2}{8 . b} \cdot \left(\frac{1}{k} + \frac{15}{2} \cdot \frac{z_1}{k^2} \right) \mp \frac{1}{2} \cdot \frac{a . t . E . I_1}{q . z_1 . k . r} + \frac{w' . l^2}{q . z_1} \cdot \left(\frac{r^2}{6} - \frac{r^3}{6} + \frac{r^4}{20} \right) \tag{61}$$

Hence, p'_1, the *maximum thrust* $= \frac{1}{A_1}\cdot\left(H_1 + \frac{M_1}{q.z_1}\right)$

$$= \frac{1}{A_1}\left\{\frac{w.l^2}{8}\cdot\frac{\frac{1}{k} + \frac{15}{2}\cdot\frac{z_1}{k^2}}{1 + \frac{45}{4}\cdot q\cdot\frac{z_1^2}{k^2}} \mp \frac{1}{2}\cdot\frac{a.t.E.I_1}{q.z_1.k.r} + \frac{w'.l^2}{6.q.z_1}\cdot\left(r^2 - r^3 + \frac{3}{10}\cdot r^4\right)\right\} \tag{62}$$

Similarly, p''_1, the *maximum tension* $= \frac{1}{A_1}\left(-H_1 + \frac{M_1}{q.z_1}\right)$

$$= \frac{1}{A_1}\left\{\frac{w.l^2}{8}\cdot\frac{\frac{15}{2}\cdot\frac{z_1}{k^2} - \frac{1}{k}}{1 + \frac{45}{4}q\cdot\frac{z_1^2}{k^2}} \pm \frac{1}{2}\cdot\frac{a.t.E.I_1}{q.z_1.k.r} + \frac{w'.l^2}{6q.z_1}\cdot\left(r^2 - r^3 + \frac{3}{10}\cdot r^4\right)\right\} \tag{63}$$

Corollary 2.—Let the depth of the rib be small compared with k.

$\therefore \frac{z_1}{k}$ is a small fraction, and the maximum value of p_1 both for tension and compression corresponds to $r = \frac{2}{5}$, approximately.

Hence, equations (62) and (63), respectively become,

$$p'_1 = \frac{1}{A_1}\cdot\left\{\frac{w.l^2}{8}\cdot\left(\frac{1}{k} + \frac{15}{2}\cdot\frac{z_1}{k^2}\right) \mp \frac{5}{4}\cdot\frac{a.t\,E.I_1}{q.z_1.k} + \frac{54}{3125}\cdot\frac{w'.l^2}{q.z_1}\right\} \tag{64}$$

$$p''_1 = \frac{1}{A_1}\cdot\left\{\frac{w.l^2}{8}\cdot\left(-\frac{1}{k} + \frac{15}{2}\cdot\frac{z_1}{k^2}\right) \mp \frac{5}{4}\cdot\frac{a.t.E.I_1}{q.z_1.k} + \frac{54}{3125}\cdot\frac{w'.l^2}{q.z_1}\right\} \tag{65}$$

Corollary 3.—If $2\,b > 5.c$, then r must be *unity*.

Equations (42), (46), (62) and (63), in this case respectively become,

$$H = \frac{l^2}{8}\cdot\frac{w + w'}{k.b} \pm \frac{15}{8}\cdot\frac{a.t}{b}\cdot\frac{E.I_1}{k^2} \tag{66}$$

$$M_1 = -M_o = \frac{l^2}{12}\cdot\frac{w + w'}{b}\cdot\frac{45}{4}\cdot q\cdot\frac{z_1^2}{k^2} \pm \frac{5}{4}a.t\,\frac{E.I_1}{b.k} = \frac{15}{16}\cdot l^2\cdot\frac{w+w'}{b}\cdot q\cdot\frac{z_1^2}{k^2} \pm \frac{5}{4}\cdot\frac{a.t}{b}\cdot\frac{E.I_1}{k} \tag{67}$$

$$p'_1 = \frac{1}{A_1}\cdot\left\{\frac{w.l^2}{8}\cdot\frac{\frac{1}{k} + \frac{15}{2}\cdot\frac{z_1}{k^2}}{b} \mp \frac{1}{2}\cdot\frac{a.t.E.I_1}{q.z_1.k} + \frac{1}{20}\cdot\frac{w'.l^2}{q.z_1}\right\} \tag{68}$$

$$p''_1 = \frac{1}{A_1}\left\{\frac{w.l^2}{8}\cdot\frac{-\frac{1}{k} + \frac{15}{2}\cdot\frac{z_1}{k^2}}{b} \pm \frac{1}{2}\cdot\frac{a.t.E.I_1}{q.z_1.k} + \frac{1}{20}\cdot\frac{w'.l^2}{q.z_1}\right\} \tag{69}$$

Corollary 4.—A nearer approximation than is given by the preceding results may be obtained as follows:—

Let $x + dx$, $y + dy$, be the co-ordinates of a point very near C *before* deformation.

Let $x' + dx'$, $y' + dy'$, be the co-ordinates of a point very near *C after* deformation.

$$\therefore ds^2 = dx^2 + dy^2 \text{ and } ds'^2 = dx'^2 + dy'^2$$

$$\therefore ds'^2 - ds^2 = dx'^2 - dx^2 + dy'^2 - dy'^2$$

or, $(ds' - ds).\ (ds' + ds) = (dx' - dx).\ (dx' + dx) + (dy' - dy)(dy' + dy)$

$\therefore\ (ds' - ds).ds = (dx' - dx).dx + (dy' - dy)dy$, approximately.

$$\therefore dx' - dx = (ds' - ds).\frac{ds}{dx} - (dy' - dy).\frac{dy}{dx}$$

$$\text{and } dy' - dy = (ds' - ds)\frac{ds}{dx}.\frac{dx}{dy} - (dx' - dx).\frac{dx}{dy}$$

Hence, by Equations (6) and (7),

$$dx' - dx = .\Delta\theta\frac{dy}{dx}.dx - \frac{T}{E.A}.\left(\frac{ds}{dx}\right)^2.dx \pm a.t.\left(\frac{ds}{dx}\right)^2.dx$$

$$\text{and } dy' - dy = -\Delta\theta.dx - \frac{T}{E.A}.\left(\frac{ds}{dx}\right)^2.\frac{dx}{dy}.dx \pm a.t.\left(\frac{ds}{dx}\right)^2.\frac{dx}{dy}.dx$$

$$\therefore x' - x = \int_0^x \Delta\theta.\frac{dy}{dx}.dx - \int_0^x \frac{T}{E.A}.\left(\frac{ds}{dx}\right)^2.\ dx \pm a.t.\int_0^x \left(\frac{ds}{dx}\right)^2.dx$$

$$\text{and } y' - y = -\int_0^x \Delta\theta.dx - \int_0^x \frac{T}{E.A}.\left(\frac{ds}{dx}\right)^2.\frac{dx}{dy}.dx \pm a.t.\int_0^x \left(\frac{ds}{dx}\right)^2.\frac{dx}{dy}.dx$$

These equations are to be used instead of equations (8) and (9), the remainder of the calculations being computed precisely as before.

(14). *Parabolic Rib of uniform stiffness, supported at the ends, but not fixed.*—

Let the rib be similar to that of the preceding article, but let its ends be free. In this case $M_o = M_1 = 0$, while $\Delta\theta$ is an undetermined constant.

The following equations apply:—

$$(A)\quad S = S_o + \left(\frac{8.k.H}{l^2} - w\right).x \qquad (26')$$

$$(B)\quad S = S_o + \left(\frac{8.k.H}{l^2} - w\right)x - w'.\{x - (1-r).l\} \qquad (27')$$

$$(A)\quad M = S_o.x + \left(\frac{8.k.H}{l^2} - w\right).\frac{x^2}{2} \qquad (28')$$

$$(B)\quad M = S_o.x + \left(\frac{8.k.H}{l^2} - w\right).\frac{x^2}{2} - \frac{w'}{2}.\{x - (1-r).l\}^2 \qquad (29')$$

$$(A)\quad \Delta\theta = \Delta\theta_o - \frac{1}{E.I}.\left\{S_o.\frac{x^2}{2} + \left(\frac{8.k.H}{l^2} - w\right).\frac{x^3}{6}\right\} \qquad (31')$$

$$(B)\quad \Delta\theta = \Delta\theta_o - \frac{1}{E.I}\left[S_o.\frac{x^2}{2} + \left(\frac{8.k.H}{l^2} - w\right).\frac{x^3}{6} - \frac{w'}{6}.\{x - (1-r)l\}^3\right] \qquad (32')$$

Assume that the horizontal and vertical displacements of the loaded end are *nil*. Substitute the value of $\Delta\,\theta$ from (32′) in (35) and (36); integrate and reduce, neglecting the term involving the temperature.

$$\therefore 0 = \Delta\theta_o - \frac{1}{E.I_1}.\left\{S_o.\frac{l^3}{12} + \left(\frac{8.k.H}{l^2} - w\right).\frac{l^3}{60} - w'.l^3.\frac{r^5}{60}\right\} - \frac{H}{4}.\frac{1}{E.A_1}.\frac{l}{k} \quad (37')$$

$$\text{and } 0 = \Delta\,\theta_o - \frac{1}{E.I_1}\left\{S_o.\frac{l^2}{6} + \left(\frac{8.k.H}{l^2} - w\right).\frac{l^3}{24} - w'.l^3.\frac{r^4}{24}\right\} \quad (38')$$

From (29′), since $M_1 = 0$,

$$\therefore 0 = S_o + \left(\frac{8.k.H}{l^2} - w\right).\frac{l}{2} - w'.l.\frac{r^2}{2} \quad (70)$$

Equations (37′), (38′), and (70), are the equations of condition.

Subtract (38′) from (37′),

$$\therefore 0 = \frac{1}{E.I_1}.\left\{S_o.\frac{l^3}{12} + \left(\frac{8.k.H}{l^2} - w\right).\frac{l^3}{40} - w'.l^3.\left(\frac{r^4}{24} - \frac{r^5}{60}\right)\right\} - \frac{H}{4}.\frac{1}{E.A_1}.\frac{l}{k}$$

which may be written,

$$0 = S_o + \left(\frac{8.k.H}{l^2} - w\right).\frac{3.l}{10} - w'.l.\left(\frac{r^4}{2} - \frac{r^5}{5}\right) - 3.H.\frac{I_1}{A_1}.\frac{1}{k.l} \quad (71)$$

Subtract (71) from (70),

$$\therefore 0 = \left(\frac{8\,k.H}{l^2} - w\right).\frac{l}{5} - w'.l.\left(\frac{r^2}{2} - \frac{r^4}{2} + \frac{r^5}{5}\right) + 3.H.\frac{I_1}{A_1}.\frac{1}{k.l} \quad (72)$$

$$\text{Hence, } H = \frac{l^2.\left\{w + \frac{w'}{2}.(5.r^2 - 5.r^4 + 2.r^5)\right\}}{8.k.\left(1 + \frac{15}{8}.\frac{I_1}{A_1}.\frac{1}{k^2}\right)} \quad (73)$$

Eliminating S_o between (38′) and (70),

$$\therefore \Delta\,\theta_o = -\frac{1}{E.I}.\left\{\left(\frac{8.k.H}{l^2} - w\right).\frac{l^3}{24} - w'.l^3.\left(\frac{r^2}{12} - \frac{r^4}{24}\right)\right\} \quad (74)$$

Also, by (27′),

$$S_1 = S_o + \left(\frac{8.k.H}{l^2} - w\right).l - w'.r.l = -P, \text{ suppose.} \quad (75)$$

Eliminating S_o between (70) and (75),

$$\therefore\ -P = S_1 = \left(\frac{8.k.H}{l^2} - w\right).\frac{l}{2} - w'.l.\left(r - \frac{r^2}{2}\right) \quad (76)$$

(70), (73), (74) and (76) give the values of H, S_o, S_1, and .

Again, the maximum bending moment M' occurs at a point given by

$$\frac{dM}{dx} = 0 \text{ in } (29').$$

$$\therefore\ 0 = S_o + \left(\frac{8.k.H}{l^2} - w\right).x - w'.\{x - (1 - r).l\} \quad (77)$$

Subtract (77) from (75),

$$\therefore\ -P_1=S_1=\left(\frac{8.k.H}{l^2}-w\right)\cdot(l-x)\ -w'.(l-x)$$

Hence, the distance from the loaded end of the point at which the bending moment is greatest is,

$$l-x=\frac{P}{w+w'-\dfrac{8.k.H}{l^2}} \tag{78}$$

Substitute this value of x in (29′), and, for convenience, put

$$w+w'\ -\frac{8.k.H}{l^2}=m.$$

$$\therefore M'=S_o.\left(l-\frac{P}{m}\right)+\frac{w'-m}{2}\cdot\left(l-\frac{P}{m}\right)^2-\frac{w'}{2}\cdot\left(-\frac{P}{m}+r.l\right)^2$$

$$=l.\left(S_o+\frac{w'-m}{2}\cdot l-\frac{w'}{2}\cdot r^2.l\right)-\frac{P}{m}\cdot(S_o+\overline{w'-m}.l-w'.r.l)+\frac{P^2}{m^2}\cdot\left(\frac{w'-m}{2}-\frac{w'}{2}\right)$$

But by (70), $0=S_o+\frac{w'-m}{2}\cdot l-\frac{w'}{2}\cdot r^2.l$

$$\therefore\ M'=l.(0)-\frac{P}{m}(-P)+\frac{P^2}{m^2}\cdot\left(-\frac{m}{2}\right)=\frac{P^2}{2.m}$$

Hence, M', the *maximum bending moment*, $=\dfrac{P^2}{2.\left(w+w'-\dfrac{8.k.H}{l^2}\right)}$ (79)

As before the greatest stress (a *thrust*) $=\frac{1}{A_1}\cdot\left(H+\frac{A_1.z_1}{I_1}.M'\right)=p'_1$, (80) and the value of r which makes p'_1 an *absolute* maximum is given by $\frac{dp'_1}{dr}=0$. But by (79), M' involves r^{10} in the numerator, and r^5 in the denominator, so that $\frac{dp'_1}{dr}=0$, will be an equation involving r^{14}.

One of its roots is $r=1$, which generally gives a *minimum* value of p'_1. Dividing by $r-1$, the equation reduces to one of the *thirteenth* order, but is still far too complex for use. It is found, however, that $r=\frac{1}{2}$ gives a *close approximation* to the *absolute* maximum thrust.

With this value of r, and, for convenience, putting $1+\frac{15}{8}\cdot\frac{I_1}{A_1}\cdot\frac{1}{k^2}=n$,

By (73), $$H=\frac{l^2}{8.k.n}\cdot\left(w+\frac{w'}{2}\right), \tag{81}$$

By (70), $$S_o=\frac{l}{2}\cdot\left\{\left(w+\frac{w'}{2}\right)\frac{n-1}{n}-\frac{w'}{4}\right\}, \tag{82}$$

By (76), $$-S_1=P=\frac{l}{2}\cdot\left\{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}+\frac{w'}{4}\right\}, \tag{83}$$

By (74), $$\Delta\theta_o=\frac{l^3}{24.E.I}\left\{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}-\frac{w'}{16}\right\}. \tag{84}$$

By (78), $$l-x=\frac{\frac{l}{2}\cdot\left\{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}+\frac{w'}{4}\right\}}{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}+\frac{w'}{2}} \tag{85}$$

By (79) $$M'=\frac{\frac{l^2}{8}\cdot\left\{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}+\frac{w'}{4}\right\}^2}{\left(w+\frac{w'}{2}\right)\cdot\frac{n-1}{n}+\frac{w'}{2}} \tag{86}$$

(15).—*Parabolic Rib of uniform stiffness hinged at the crown and also at the ends.*—In this case $M=0$ at the crown, which introduces a *fourth* equation of condition.

By (29′), $$0=S_o.\frac{l}{2}+\left(\frac{8k.H}{l^2}-w\right)\cdot\frac{l^2}{8}-\frac{w'.l^2}{2}\cdot\left(-\frac{1}{2}+r\right)^2$$

which may be written,

$$0=S_o+\left(\frac{8.k.H}{l^2}-w\right)\cdot\frac{l}{4}-w'.l.\left(r^2-r+\frac{1}{4}\right) \tag{87}$$

Eliminating S_o between (85) and (70),

$$\therefore\frac{8.k.H}{l^2}-w=w'.(-2r^2+4r-1)$$

Hence, $$H=\frac{l^2}{8.k}\cdot\left\{w-w'.(2r^2-4r+1)\right\} \tag{88}$$

By (87), $$S_o=\frac{w'.l}{2}.(3r^2-4r+1) \tag{89}$$

By (76), $$P=S_1=\frac{w'.l}{2}.(r-1)^2 \tag{90}$$

By (74), $$\Delta\theta_o=\frac{w'.l^3}{24.E.I_1}(1-4.r+4.r^2-r^4) \tag{91}$$

By (78) and (90), $$l-x=\frac{\frac{w'.l}{2}.(r-1)^2}{2.w'(r-1)^2}=\frac{l}{4} \tag{92}$$

By (79), $$M'=\frac{w'.l^2}{16}.(r-1)^2 \tag{93}$$

When $r=\frac{1}{2}$,

$$\left.\begin{array}{l} \therefore H=\frac{l^2}{8.k}\cdot\left(w+\frac{w'}{2}\right)\cdot S_o=-\frac{w.l}{8},\ P=-S_1=\frac{w'.l}{8} \\ \Delta\theta_o=-\frac{1}{384}\cdot\frac{w'.l^3}{E.I_1},\ \text{and}\ M'=\frac{w'.l^2}{64} \end{array}\right\} \quad (94)$$

These results agree with those of (81) to (86), if $n=1$.

In general, when $n=1$,

$$w+\frac{w'}{2}(5.r^3-5.r^4+2.r^5)=w-w'\,(2.r^3-4.r+1),\ \text{by (73) and (88)}$$

$$\therefore 2.r^5-5.r^4+9.r^3+8.r+2=0=(2.r-1).(r-1)^2.(r^2-2),$$

and the roots are, $r=\frac{1}{2}$, $r=1$, $r=\pm\sqrt{2}$.

Hence, $n=1$ only renders the expressions in (94) identical with the corresponding expressions of the preceding article when $n=\frac{1}{2}$ or 1.

Again, the intensity of thrust is greatest at the outer flange of the loaded and the inner flange of the unloaded half of the rib, and is

$$=\frac{l^2}{8.A_1}\cdot\left\{\frac{z_1}{I_1}\cdot\frac{w'}{8}+\frac{1}{k}\cdot\left(w+\frac{w'}{2}\right)\right\}$$

The intensity of tension is greatest at the inner flange of the loaded and the outer flange of the unloaded half of the rib, and is

$$=\frac{l^2}{8.A_1}\left\{\frac{z_1}{I_1}\cdot\frac{w'}{8}-\frac{1}{k}\cdot\left(w+\frac{w'}{2}\right)\right\}$$

The *greatest total horizontal thrust* occurs when $r=1$, and its value is

$$\frac{l^2}{8.k}(w+w')$$

(16).—*Maximum deflection of an arched rib.*—The deflection must necessarily be a maximum at a point given by $\Delta\,\theta=0$. Solve for x and substitute in (9) to find the deflection $y'-y$; the deflection is an *absolute* maximum when $\frac{d}{dr}.(y'-y)=0$. The resulting equation involves r to a high power, and is too intricate to be of use. It has been found by trial, however, that in all ordinary cases the absolute maximum deflection occurs at the middle of the rib, when the live load covers its whole length, *i.e.*, when $x=\frac{l}{2}$, and $r=1$.

Case I.—*Rib of* §(13).—For convenience, put $1+\frac{45}{4}\cdot\frac{I_1}{A.k^2}=s$

$$\therefore \text{By (42)},\qquad H=\frac{l^2}{8.k}\cdot\frac{w+w'}{s}\pm\frac{15}{8}\cdot\frac{a.t}{s}\cdot\frac{E.I_1}{k^2} \qquad (95)$$

By (43) and (46), $-M_o = \frac{l^2}{12}\cdot(w+w').\frac{s-1}{s} \mp \frac{5}{4}\cdot\frac{a.t}{s}\frac{E\,I_1}{k} = -M_1$ (96)

By (38) and (43), $S_o = -6.\frac{M_o}{l}$ (97)

By (32), (43), (97), $\Delta\,\theta = -\frac{1}{E.I_1}\cdot\left(M_o.x - 3.M_o.\frac{x^2}{l} + 2.M_o.\frac{x^3}{l^2}\right)$ (98)

Hence, the maximum deflection

$$= -\int_{\frac{l}{2}}^{o}\Delta\,\theta.dx = -\frac{M_o}{E.I}\int_{o}^{x}\left(x - 3.\frac{x^2}{l} + 2.\frac{x^3}{l^2}\right)\cdot dx = -\frac{M_o}{E.I}\cdot\frac{l^2}{32}$$

$$= \frac{l^4}{384}\cdot\frac{w+w'}{E.I_1}\cdot\frac{s-1}{s} \mp \frac{5}{128}\cdot\frac{a.t}{s}\cdot\frac{l^2}{k} = d_1, \text{ suppose.} \quad (99)$$

The central deflection d_2 of a uniform straight horizontal beam of same span, of same section as the rib at the crown, and with its ends fixed, is,

$$d_2 = \frac{l^4}{384}\cdot\frac{w+w'}{E.I_1} \quad (100)$$

Hence, neglecting the term involving the temperature,

$$d_1 = \frac{s-1}{s}\cdot d_2 \quad (101)$$

CASE II.—*Rib of* §(14).—

By (73), $H = \frac{l^2}{8.k}\cdot\frac{w+w'}{n}$ (102)

By (74) and (70), $\Delta\theta_o = \frac{l^3}{24.E.I_1}\cdot(w+w').\frac{n-1}{n} = \frac{S_o.l^2}{12.E.I}$ (103)

By (32), (102) and (103), $\Delta\theta = \frac{S_o}{E.I_1}\cdot\left(\frac{l^2}{12} - \frac{x^2}{2} + \frac{x^3}{3.l}\right)$ (104)

Hence, the maximum deflection

$$= \frac{S_o}{E.I_1}\int_{o}^{\frac{l}{2}}\left(\frac{l^2}{12} - \frac{x^2}{2} - \frac{x^3}{3.l}\right).dx = \frac{5}{384}\cdot\frac{l^4}{E.I}\cdot(w+w').\frac{n-1}{n} = d'_1 \quad (105)$$

If the ends of the beam in Case I are free, its central defection

$$= \frac{5}{384}\cdot\frac{l^4\,(w+w')}{E.I} = d'_2,$$

$$\therefore\ d'_1 = \frac{n-1}{n}\cdot d'_2 \quad (106)$$

Thus, the deflection of the arched rib in both cases is less than that of the beam.

(17).—*Arched rib of uniform stiffness fixed at the ends and connected at the crown with a horizontal distributing girder* (See § (2).—The load is transmitted to the rib by vertical struts so that the vertical displacements of corresponding points of the rib and girder are the same. The horizontal thrust in the loaded is not necessarily equal to that in the unloaded division of the rib, but the excess of the thrust in the loaded division will be borne by the distributing girder, if the rib and girder are connected in such a manner that the horizontal displacement of each at the crown is the same.

The formulæ of § (13) are applicable in the present case with the modification that I_1 is to include the moment of the inertia of the girder.

The maximum thrust and tension in the rib are given by Equations (64) and (65).

Let z' be the depth of the girder, A' its sectional area,

$$\text{The greatest thrust in the girder} = \frac{H}{A_1 + A'} + \frac{M_1 . z'}{2.E.I_1} \qquad (107)$$

$$\text{“} \qquad \text{tension} \qquad \text{“} \qquad = \frac{M_1 . z'}{2.E.I_1} \qquad (108)$$

H and M_1 being given by Equations (66) and (67), respectively.

The girder must have its ends so supported as to be capable of transmitting a thrust.

(18).—*Semi-circular timber rib, hinged at the ends.*

Let $P\ C\ Q$ be the axis of such a rib. Consider a portion CC'' bounded by vertical planes at C and C''.

Let x, y be the co-ordinates of C'' with respect to C.

Let r be the radius of the semi-circle.

Let w be the intensity of the load which is assumed to be *uniformly distributed horizontally.*

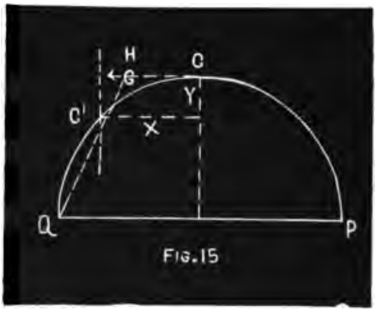


FIG. 15

The moment of resistance of the rib at C''=the algebraic sum of the moments of the thrust at C, and of the weight upon CC'', with respect to C'.

$$\therefore M = -H.y + \frac{w.x^2}{2}$$

At either end, $x = y = r$, and $M = 0$. $\therefore H = \frac{w.r}{2}$

also $x^2 = y.\ (2.r - y)$

$$\therefore M = -\frac{w.r}{2} y + \frac{w}{2} y.\ (2.r - y) = \frac{w}{2} y.(r - y)$$

M is a maximum when $y = \frac{r}{2}$, its value then being $\frac{w.r^2}{8}$

In the horizontal tangent at C, take $CG = \frac{r}{2}$; $Q\ G$ will be the direction of the resultant thrust at Q.

(19).—*Stresses in spandril posts and diagonals.*

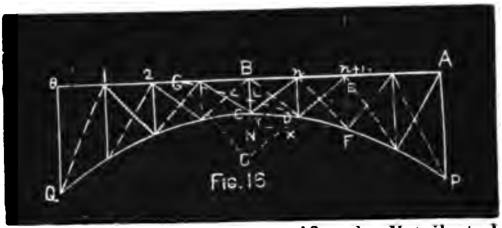


Fig. 15

Fig. 16 represents an arch in which the spandril consists of a series of vertical posts and diagonal braces.

Let the axis of the curved rib be a parabola. The arch is then equilibrated under a uniformly distributed load, and the diagonals will be only called into play under a passing load.

Let x, y, be the co-ordinates of any point F of the parabola with respect to the vertex C. $\therefore y = \frac{4.k}{l^2} \cdot x^2$

Let the tangent at F meet $C\,B$ in L, and the horizontal $B\,E$ in G.

Let $B\,C = k'$ $\therefore B\,L = B\,C - C\,L = B\,C - C\,N = k' - y$.

Let N be the total number of panels.

Consider any diagonal $E\ D$ between the n-th and $(n + 1)$-th posts.

Let w' be the greatest panel *live* load.

The greatest compression in $E\,D$ occurs when the passsng load is concentrated at the first n–1 panel points.

Imagine a vertical section a little on the left of EF.

The portion of the frame on the right of this section is kept in equilibrium by the reaction R at P and by the stresses in the three members met by the secant plane.

Take moments about G.

$$\therefore D.GE.\cos\theta = R.AG,$$

D being the stress in DE, and θ the angle DEF.

$$\text{Now, } R=\frac{w'}{2}\cdot\frac{n.n-1}{N}$$

$$\text{also, } \frac{x+GB}{GB}=\frac{k'+y}{k'-y},\ \therefore GB=\frac{k'.x-x.y}{2.y}$$

$$\text{and } \therefore\ GE=GB+x=\frac{k'.x+x.y}{2.y}, \text{ and } GA=\frac{l}{2}+\frac{k'.x-x.y}{2.y}$$

$$\text{Hence, } D=\frac{w'}{2}\cdot\frac{n.n-1}{N}\cdot\frac{ly+k'.x-x.y}{k'.x+x.y}.\sec\theta$$

The stresses in the counter braces, (shewn by dotted lines in the Fig.), may be obtained in the same manner.

The greatest thrust in $EF=w'+w$

" " tension " $=D.\cos\theta-w$,

w being the *dead* load upon EF.

If the last expression is negative, EF is never in tension.

(20)—*Clerk Maxwell's method of determining the resultant thrusts at the supports of a framed arch.*

Let Δs be the change in the length s of any member of the frame under the action of a force P, and let a be the sectional area of the member.

$\therefore \pm\frac{P}{E.a}.s=\Delta s$, the sign depending upon the character of the stress.

Assume that all the members, except the one under consideration, are perfectly rigid, and let Δl be the alteration in the span l corresponding to Δs. The ratio $\frac{\Delta l}{\Delta s}$ is equal to a constant m, which depends only upon the geometrical form of the frame.

$$\therefore \Delta l=m.\Delta s=\pm m.P.\frac{s}{E.a}$$

Again, P may be supposed to consist of two parts, viz., f_1 due to a horizontal force H between the springings, and f_2 due to a vertical force V applied at one springing, while the other is firmly secured to keep the frame from turning.

By the principle of virtual velocities,

$$\frac{f_1}{H}=\frac{\Delta l}{\Delta s}=m$$

Similarly, $\frac{f_2}{V}$ is equal to some constant n, which depends only upon the *form* of the frame.

$$P=f_1+f_2=m.H+n.V$$

$$\therefore \Delta l=\pm(m^2.H+m.n.V)\cdot\frac{s}{E.a}$$

Hence, the total change in l for all the members is,

$$\Sigma.\Delta l = \pm\Sigma\left(m^2 H.\frac{s}{E.a}\right) \pm \Sigma\left(m.n.V.\frac{s}{E.a}\right)$$

If the abutments yield, let $\Sigma\Delta l = \mu.H$, μ being some co-efficient to be determined by experiment.

$$\therefore H = \frac{\pm\Sigma\left(m.n.V.\frac{s}{E.a}\right)}{\mu \mp \Sigma\left(m^2.\frac{s}{E.a}\right)} \qquad \text{(A)}$$

If the abutments are immovable, $\Sigma.\Delta l$ is *zero*.

$$\text{and } H = -\frac{\Sigma\left(m.n.V.\frac{s}{E.a}\right)}{\Sigma\left(m^2.\frac{s}{E.a}\right)} \qquad \text{(B)}$$

V is the same as the corresponding reaction at the end of a girder of the same span, and similarly loaded. The required thrust is the resultant of H and V, and the stress in each member may be computed graphically or by the method of moments. In any particular case proceed as follows:—

(1).–Prepare tables of the values of m and n for each member.

(2).–Assume a cross-section for each member, based on a probable assumed value for the resultant of V and H.

(3).–Prepare a table of the value of $m^2.\frac{s}{E.a}$ for each member, and form the sum $\Sigma\left(m^2.\frac{s}{E.a}\right)$.

(4).–Determine, separately, the horizontal thrust between the springings due to the loads at the different joints. Thus, let v_1, v_2 be the vertical reactions at the right and left supports due to any *one* of these loads. Form the sum $\Sigma\left(m.n.V.\frac{s}{E.a}\right)$ using v_1 for all the members on the right of the load and v_2 for all those on its left. The corresponding thrust may then be found by Equation (A) or Equation (B), and the *total* thrust H is the sum of the thrusts due to all the weights taken separately.

(5).–Repeat the process for each combination of live and dead load so as to find the maximum stresses to which any member may be subjected. (§7).

(6).–If the assumed cross-sections are not suited to these maximum stresses, make fresh assumptions, and repeat the whole calculation.

Note.–The same method may be applied to determine the resultant tensions at the supports of a framed suspension bridge.

Note.—The formulæ for a parabolic rib may be applied without material error to a rib in the form of a segment of a circle. More exact formulæ may be obtained for the latter in a manner precisely similar to that described in Articles 13 to 17, but the integrations will be much simplified by using polar coordinates, the centre of the circle being the pole.

CHAPTER VI.

DETAILS OF CONSTRUCTION.

(1).—*Chords of main trusses and floor beams.*—The chords of a main truss are to be designed for a uniform distribution of the live load over the whole of the bridge.

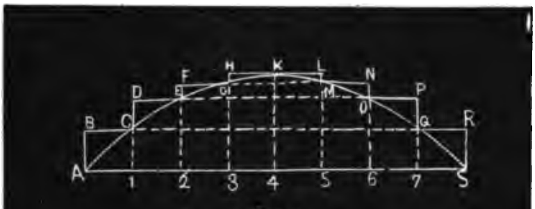


FIG 1

First, let the depth of the truss be uniform throughout. Draw the curve of bending moments A C E G K M O Q S to a given scale. Any ordinate of the same curve may, by altering the scale, be made to represent the thickness of the chord at its foot.

Erect ordinates 1 C, 2 E, 3 G...at each panel point, and complete the rectangles BS, DQ, FO, and HM. The broken outline A B C D E F G H L M O P Q R S is a delineation of the least admissable thickness of the chord at different points in its length.

The floor-beams of a highway bridge are subjected to a distribution of load which is approximately uniform, and may be designed in the same manner.

Theoretically, the thickness of the chord at the ends is zero, but in practice it is made equal to that at the nearest panel point.

Next, let the depth of the truss or beam vary. Draw the bending moment curve to such a scale that its greatest ordinate coincides with the central depth. If this curve be taken as the axis of the chord, the thickness of the latter is constant.

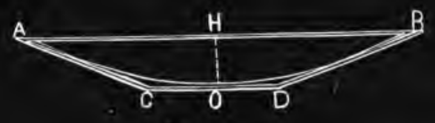


FIG 2

Let $A\ O\ B$ be the bending moment curve for the tension flange of a floor-beam for a highway bridge, the central depth of the beam being $O\ H$ (Fig. 2). $A\ O\ B$ is the axis of the curved flange, but, in order to avoid the difficulty of curving, the flange is often constructed in three straight pieces AC, CD, DB, tangential to the curve.

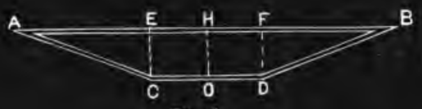


FIG. 3

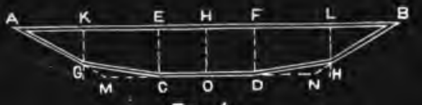


FIG. 4

In *railway* bridges, the live load is concentrated at single points of the floor beams. Thus, for a *single* track, the bending moment curve is the broken line $ACDB$ (Fig. 3), the load being applied at the two points E and F, while for a *double* track the curve is the broken line $A\ G\ C\ D\ H\ B$, the load being applied at the four points K, E, F, and L, (Fig. 4).

As before, if $H\ O$ is the central depth of the beam, the broken line in each Fig. is the axis of the corresponding tension flange, which may therefore be made of *three* straight pieces for the *single*, and of *five* for the double track. In the latter case, however, the flange is generally made of the three straight pieces, AM, MN, NB.

The depth of a floor-beam may vary from $\frac{1}{15}$-th to $\frac{1}{8}$-th of its clear span, and should be made as great as possible.

(2).—*Platform.*—The floor of a *highway* bridge is laid upon a series of stringers, which run parallel to the centre line of the bridge, are spaced 2-ft. centre to centre, and are supported by the floor-beams. The latter are consequently loaded at several equi-distant points, and the distribution of the load may be assumed to be approximately uniform. In

U CALIFORNIA

a railway bridge a stringer is usually placed underneath each line of rails, and is supported by the floor-beams, which may, therefore, be loaded at *two* or *four* points according to the number of the tracks.

The entire platform may be constructed of timber wherever such material is abundant. The floor of a narrow bridge, for example, may consist of strong planks stretching between the main trusses, or, better still, the planks may be laid upon timber floor-beams. These floor-beams, for a single-track bridge, are often 12-ins. wide by 9-ins. deep, and are spaced about 3-ft. centre to centre. The rails are laid upon longitudinal stringers which are about 12 ins. wide by 9-ins. deep, and are notched and spiked to the floor-beams. In all such cases the main trusses should be braced together with iron tie-rods.

A timber platform may be set on fire by the cinders from a locomotive, and it is sometimes necessary to protect the surface with iron plates, gravel, etc. If there is any traffic underneath the bridge the planks should be tongued, and the joints caulked, so as to prevent leakage.

The entire roadway is rarely made to depend upon timber only, except in the case of small bridges. The longitudinals and floor-beams may be of wrought-iron or steel, and are nearly always *I*-shaped in section. Rolled joists, from 4-ins. deep and 18-lbs. per lineal yd. to 15-ins. deep and 200-lbs. per yd., may be obtained in the market at a reasonable cost, and are recommended as floor-beams. In railway bridges, however, built plate-beams are more common, as they can be connected to the main trusses with greater ease and effect. In designing these plate-beams, the dimensions of the plates should be kept as uniform as is practicable, and should be such as are obtainable in the market, special sizes invariably adding to the cost.

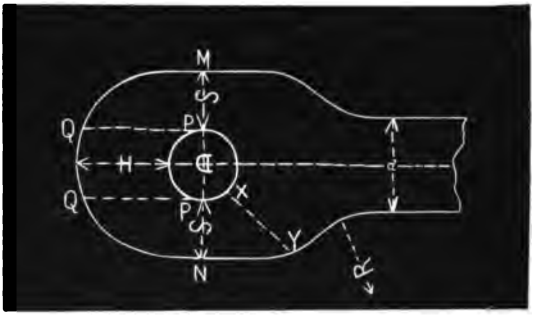
The floor is often made of timber for the sake of economy and lightness, but a floor of curved, corrugated, buckled, or flat plates stiffened with *angles* or *tees*, will add to the strength and durability of the whole bridge.

Cast-iron floor-beams have been rarely adopted except in cast-iron bridges.

(3).—*Pins.*—One of the most important members of a bridge constructed on the pin-system is the *eye-bar*, or *link*, which is a flat bar with enlarged ends, having eyes for the insertion of bolts or pins. The eyes are forged in the ends under hydraulic pressure, with suitably shaped dies. Careful mathematical and experimental investigations have been undertaken to determine the proper dimensions of the link-head and pin.

In consequence of the complex character of the stresses developed, an *exact* mathematical solution is impossible, but approximate results may be obtained which differ very little from the results of experiment.

Let d be the depth, and t the thickness of the *shank* of the eye-bar represented in Fig. 5. Let S be the width of the metal at the sides of the eye, and H the width at the end. Let D be the diameter of the pin.



The proportions of the *head* are governed by the general condition that each and every part should be at least as strong as the shank.

When the bar is subjected to a tensile stress the pin is tightly embraced, and failure may arise from any one of the following causes:—

(*a*).–*The pin may be shorn through.*

Hence, if the pin is in double shear, its sectional area should be at least *one-half* that of the shank.

It may happen that the pin is bent, but that fracture is prevented by the closing up of the pieces between the pin-head and nut; the efficiency, however, of the connection is destroyed, as the bars are no longer free to turn on the pin.

In practice, D for *flat* bars, varies from $\frac{2}{3}\cdot d$ to $\frac{4}{5}\,d$ but usually lies between $\frac{3}{4}\cdot d$ and $\frac{4}{5}\cdot d$.

The diar. of the pin for the end of a round bar is generally made equal to $1\frac{1}{4}$-times the diar. of the bar.

The pin should be turned so as to fit the eye accurately, but the best practice allows a difference of from $\frac{1}{64}$ to $\frac{1}{100}$ in the diars. of the pin and eye.

(*b*).–*The link may tear across MN.*

Hence, the sectional area of the metal across MN must be at least equal to that of the shank, and in practice is always greater.

S varies from $\frac{11}{20}\cdot d$ to $\frac{5}{8}\cdot d$

The sectional area through the sides of the eye in the head of a round bar varies from 1½-times to *twice* that of the bar.

(c).–*The pin may be torn through the head.*

Theoretically, the sectional area of the metal across PQ should be *one-half* that of the shank. The metal in front of the pin, however, may be likened to a uniformly loaded girder with both ends fixed, and is subjected to a bending as well as to a shearing action. Hence, the *minimum* value of H has been fixed at $\frac{3}{5}\cdot d$, and if H is made equal to d, both kinds of action will be amply provided for.

(d).–*The bearing surface may be insufficient.*

If such be the case the intensity of the pressure upon the bearing surface is excessive, the eye becomes oval, the metal is upset, and a fracture takes place.

In practice, adequate bearing surface is obtained, by thickening the head so as to confine the maximum intensity of the pressure within a given limit.

(e).–*The head may be torn through the shoulder at XY.*

Hence, XY is made equal to d.

The radius of curvature R of the shoulder varies from $1\frac{1}{2}\cdot d$ to $7.6d$

Note.—The *thickness* of the shank should be $\frac{d}{4}$ or $\frac{2}{7}\cdot d$ at least.

(4).—*Effect of bending action upon pin.*

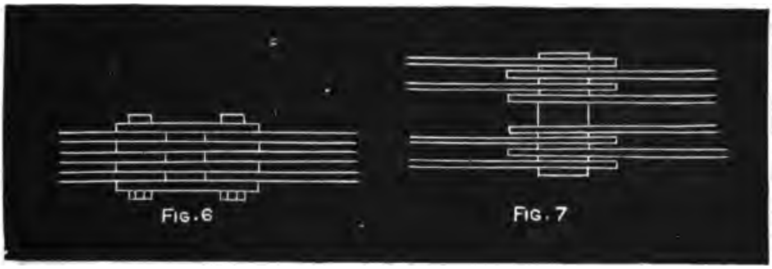


FIG. 6 FIG. 7

Figs. 6 and 7 represent groups of eye-bars as they often occur in practice.

Let two sets of $2.n$ bars pull upon the pin in opposite directions, and assume that the distribution of pressure between the pin and the eye-bar heads is uniform.

Let P be the stress in each bar.

Let p be the distance between the centre lines of two consecutive bars. All the forces on each half of the pin are evidently equivalent to a couple of which the moment is $n.P.p$.

Hence, at any section of the pin between the innermost bars,

$$n.P.p = \frac{f}{c} \cdot I,$$

f being the stress in the material of the pin at a distance c from the neutral axis, and I the moment of inertia.

If there are $2n - 2$ bars in one set, the bending action at the centre of the pin is *nil.*

In general, the bending action upon a pin connecting a number of vertical, horizontal, and inclined bars, may be determined as follows :—

Consider one-half of the pin only.

Let V, Fig. 8, be the resultant stress in the vertical bars.

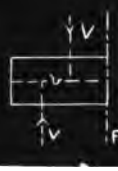


FIG. 8

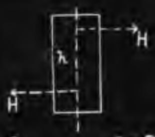


FIG. 9

It is necessarily equal in magnitude but opposite in direction to the vertical component of the resultant of the stresses in the inclined bars. Let v be the distance between the lines of action of these two resultants. The corresponding bending action upon the pin is that due to a couple of which the moment is $V.v$.

Let h be the distance between the lines of action of the equal resultants H of the horizontal stresses upon each side of the pin. The corresponding bending action upon the pin is that due to a couple of which the moment is $H.h$.

Hence, the maximum bending action is that due to a couple of which the moment is the resultant of the two moments $V.v$ and $H.h.$, viz.

$$\sqrt{(V.v)^2 + (H.h)^2}$$

(5).—*General remarks.* – Tension chord pins are most severely strained when the live load covers the whole of the bridge.

The advantages of pin-connected structures are facility of erection and transport, and the interchangeability of many of the parts. On the other hand, pins are often badly designed, the effective strength of a bar with hinged ends as a strut is considerably less than if the ends were securely fixed, and the hammering under a live load causes the eyes to elongate and the pins to work loose.

A properly riveted bridge is certainly far more rigid than one constructed on the pin system.

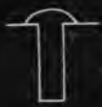


FIG. 10

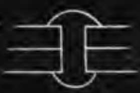


FIG. 11

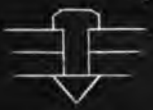


FIG. 12

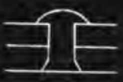


FIG. 13

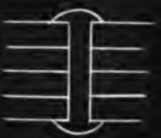


FIG. 14

(6).—*Rivets.*—A *rivet* is an iron or steel *shank*, slightly tapered at one end (the *tail*) and surmounted at the other by a *cup* or *pan-shaped head* (Fig. 10). It is used to join steel or iron plates, bars, etc. For this purpose the rivet is generally heated to a cherry red, the shank or *spindle*, is passed through the hole prepared for it, and the tail is made into a *button*, or *point*. The hollow cup-tool gives to the point a nearly hemispherical shape, and forms what is called a *snap-rivet*, (Fig. 2). Snap-rivets, partly for the sake of appearance, are commonly used in girder-work, but they are not so tight as *conical*-pointed rivets (*staff*-rivets), which are hammered into shape until almost cold, (Fig 3).

When a smooth surface is required, the rivets are *counter-sunk* (Fig. 4). The counter-sinking is drilled and may extend *through* the plate, or a shoulder may be left at the inner edge.

Cold riveting is adopted for the small rivets in boiler work and also wherever heating is impracticable, but tightly-driven turned bolts are sometimes substituted for the rivets. In all such cases the material of the rivets, or bolts, should be superior in quality.

Loose rivets are easily discovered by tapping, and, if very loose, should be at once replaced. It must be borne in mind, however, that expansions and contractions of a complicated character invariably accompany *hot-riveting*, and it cannot be supposed that the rivets will be perfectly tight. Indeed, it is extremely doubtful whether a rivet has any hold in a straight drilled hole, except at the ends.

Riveting is accomplished either by hand or machine, the latter being far the more effective. A machine will squeeze a rivet, at almost any temperature, into a most irregular hole, but the exigencies of practical conditions often prevent its use, except for ordinary work, and its advantages cannot be obtained where they would be most appreciated as, *e.g.*, in the riveting up of connexions.

(7.)—*Dimensions of rivets.*—The diameter (d) of a rivet in ordinary girder-work varies from $\frac{3}{4}$-in. to 1-inch, and rarely exceeds $1\frac{1}{8}$-in.

The thickness (t) of a plate in ordinary girder-work should never be less than $\frac{1}{4}$-in., and a thickness of $\frac{3}{8}$-in., or even $\frac{5}{16}$-in., is preferable.

Let T be the total thickness through which a rivet passes.

According to Fairbairn, when $t < \frac{1}{2}$-in., d should be about $2.t$
" " " " $t > \frac{1}{2}$-in. " " " $1\frac{1}{2}.t$
" " Unwin, when t varies from $\frac{1}{4}$-in. to 1-in. and passes through *two* thicknesses of plate, d lies between $\frac{3}{4}\cdot t + \frac{5}{16}$ and $\frac{7}{8}\cdot t + \frac{3}{8}$.
" " " when the rivets join several plates, $d = \frac{T}{8} + \frac{5}{8}$
" " French practice, diar. of head $= 1\frac{2}{3}.d$.
" " " " length of rivet from head $= T + 1\frac{2}{3}.d$
" " Rankine, length of rivet from head $= T + 2\frac{1}{2}.d$

The rise of the head $\frac{2}{3}\cdot d$

The diar. of the rivet hole is made larger than that of the shank by from $\frac{1}{32}$-in. to $\frac{1}{8}$-in., so as to allow for the expansion of the latter when hot.

There seems to be no objection to the use of long rivets, provided they are properly heated and secured.

(8).—*Strength of Punched and Drilled Plates.*—*Generally* speaking experiments indicate that the tenacity of iron and steel plates is considerably diminished by *punching* and that the effect of *drilling* is similar, though less in degree. It would seem as if the deterioration in tenacity is due, not to the cracking caused by the punch, nor to any reduction of tenacity in the neighbourhood of the hole, but rather to an alteration in the elasticity which destroys the power of elongation in a narrow annulus around the hole. The removal of the annulus (about $\frac{1}{25}$ in. thick) neutralizes the effect of punching, and the plan is therefore sometimes adopted of punching the holes $\frac{1}{8}$-in. less in diar. than the rivets, the holes being subsequently enlarged and counter-sunk.

Punching does not sensibly affect the strength of Landore unannealed plates, and only slightly diminishes the strength of thin steel plates, but causes a considerable loss of tenacity in thick steel plates; the loss, however, is less than for iron plates.

The harder the material the greater is the loss of tenacity.

Iron seems to suffer more from punching when the holes are near the edge than when removed to some distance from it, while mild steel suffers less when the hole is one diar. from the edge than when it is so far that there is no bulging at the edge.

All experiments go to prove that the original strength of a punched plate may be restored by annealing.

Whenever the metal is of an inferior quality, the holes should be drilled. Drilling is a necessity for first-class work when the diar. of the holes is less than the thickness of the plate and also when several plates are *piled*. It is impossible to punch plates, bars, angles, etc., in spite of all expedients, in such a manner that the holes in any two exactly correspond, and the irregularity becomes intensified in a pile, the passage of the rivet often being completely blocked. Recourse is then had to the barbarous *drift*, or *rimer*, which is driven through the hole by main force, cracking and bending the plates in its passage, and separating them one from another, (Fig. 5).

The holes may be punched for ordinary work, and in plates of which the thickness is less than the diar. of the rivets.

If the punch is helical or spiral in form the injury to the plate is less, and may be diminished still more by giving the hole in the die-block a diar. greater than that of the punch.

Hence, the loss of tenacity in punched plates depends upon :—

(1).–The character of the material operated upon.

(2).–The amount of the metal between the holes, or, probably, the ratio of the diar. of the punch to the pitch or the punched holes.

(3).–The diar. of the punch, or, probably, the ratio of the diar. of the punch to the thickness of the plate.

(4).–The ratio of the diar. of the punched hole to the diar. of the hole in the die-block.

(9).—*Riveted joints.*—Riveted joints may be classified as *Lap*, *Fish*, and *Butt* joints.

In *Lap* joints, the plates over-lap, (Figs. 15 and 18), and are riveted together by one or more rows of rivets.

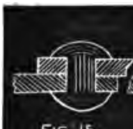


FIG. 15

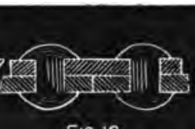


FIG. 16

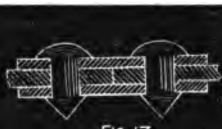


FIG. 17

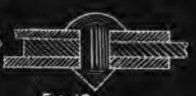


FIG. 18

In *Fish* joints, the ends of the plates meet, and the plates are riveted to a single, (Fig. 16), or to two, (Fig. 17), covers by means of one or more rows of rivets on each side of the joint.

A Fish joint is termed a *Butt* joint when the plates are in compression. The plates should butt evenly against one another, although they seldom do so in practice. Indeed, the mere process of riveting draws the plates slightly apart, leaving a gap which is often concealed by caulking. A much better method is to fill up the space with some such hard substance as cast-zinc, but the best method, if the work will allow of the increased cost, is to form a *jump joint*, *i.e.*, to plane the eyes of the plates carefully, and then bring them into close comtact, when a short cover with one or two rows of rivets will suffice to hold them in position.

The riveting is said to be *single*, *double*, *triple*,......, according as the joint is secured by *one*, *two*, *three*,......, rows of rivets.

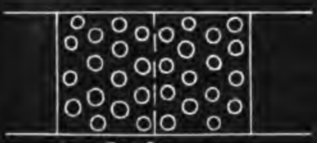


FIG. 19

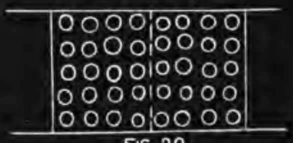


FIG. 20

Double, triple,......, riveting may be *chain*, (Fig. 19), or *zig-zag*, (Fig. 20). In the former case the rivets form straight lines longitudinally and transversely, while in the latter the rivets in each row divide the space between the rivets in adjacent rows.

(10).—*Strength of riveted joints.*—The strength of a joint, as derived from the rivets alone, evidently depends upon the number of shears to which they are liable. Thus, a rivet in *double shear*, (Figs. 17 and 18), is twice as strong as a rivet in *single shear*, (Figs. 15 and 16).

According to Fairbairn, if the strength of an unpunched plate be taken at 100, the comparative strength of single and double riveted joints are as 56 and 70, respectively, but this estimate is probably too high. At least it does not seem to be founded upon a satisfactory basis, nor to give satisfactory results, for the ratio of the tensile to the shearing stress, as deduced from Fairbairn's experiments, varies with different thicknesses of plates, which appears to be contrary to reason.

Fairbairn proposed to make the joint and unpunched plate equally strong, by increasing the thickness of the punched portion of the plate, but it is difficult to carry this out in practice.

Very few, if any, trustworthy data have been supplied respecting the resistance of plates to indentation by rivets or bolts. The resistance must of course vary with the quality of the iron or steel, but the degree in which the tenacity of the plates is affected by the indentation is most uncertain. The *mean crushing intensity* may be defined to be the ratio of the total stress to the *bearing area* (*the product of the diar. of the hole by the thickness of the plate*), and has been fixed at 5 tons per sq. in. for *wrought-iron*, but on theoretical rather than practical grounds. If it is also assumed that the bearing and tearing strengths of a joint are the same, it is found that the crushing pressure rapidly increases with the ratio of the diar. of the rivet to the thickness of the plate. When the ratio exceeds 3, the crushing pressure in a double-cover joint is more than 40,000-lbs. per sq. in., *i.e.*, it has reached its ultimate limit, and the joint will fail, although the tension in the plate is less than 8600-lbs. per sq. in. Hence, the diar. of the rivet should not be greater than *three* times the thickness of the plate. In all probability the failure of the joint at such a low tensile stress is partly due to the deformation of the metal around the hole, which causes an unequal distribution of stress.

Until further experiments give more definite information, the *safe* bearing intensity for wrought iron may be taken at 5-tons per sq. in. in ordinary work, and at $7\frac{1}{2}$-tons per sq. in. in chain rivetted joints with well supported rivets. The *safe* bearing intensity for steel may be taken at the same proportion of the ultimate stress as in the case of wrought-iron.

Let S be the total stresses at a riveted joint.

" f_1, f_2, f_3, f_4, be the safe tensile, shearing, compressive, and bearing unit stresses, respectively.

" t be the thickness of a plate, and w its width.

" N be the total number of rivets on one side of a joint.

" n " " in one row.

" p be the *pitch* of the rivets, *i.e.*, the distance centre to centre.

" d be the diar. of the rivets.

" x be the distance between the centre line of the nearest row of rivets and the edge of the plate.

It is impossible to apply the ordinary mathematical rules to the determination of the ultimate strength of a riveted joint, as the distribution of the stress which produces rupture may prove to be very irregular.

This irregularity may rise, (1).–Because the stress does not coincide in direction with the centre line, (2)–Because of local action, (3).–Because of the existence of strains within the metal before punching.

The joint may fail in any one of the following ways:—

(1).—The rivets may shear.

(2).—The rivets may be forced into and crush the plate.

(3).—The rivets may be torn out of the plate.

(4).—The plate may tear in a direction transverse to that of the stress.

The resistance to rupture should be the same in each of the four cases, and always as great as possible.

Value of x.–It has been found that the minimum safe value of x is d, and this in most cases gives a sufficient *overlap* $(=2.x)$, while $x=\frac{3}{2}\cdot d$ is a maximum limit which amply provides for the bending and shearing to which the joint may be subjected. Thus the overlap will vary from $2.d$ to $3.d$.

x may be supposed to consist of a length x_1, to resist the shearing action, and a length x_2 to resist the bending action. It is impossible to determine theoretically the exact value of x_2, as the straining at the joint is very complex, but the metal in front of each rivet (the rivets at the ends of the joint excepted) may be likened to a uniformly loaded beam of length d, depth $x_2 - \frac{d}{2}$, and breadth t, with both ends *fixed*. Its moment of resistance is therefore $\frac{f}{6}.t.\left(x_2 - \frac{d}{2}\right)^2$, f being the max. unit stress due to the bending. Also, if P is the load upon the rivet, the *mean* of the bending moments at the end and centre is $\frac{P}{8}\cdot d$.

$$\text{Hence, } \textit{approximately}, \frac{P}{8}\cdot d = \frac{f}{6}\cdot t.\left(x_2 - \frac{d}{2}\right)^2$$

$$\text{or, } P = \frac{4}{3}\cdot\frac{f.t}{d}\cdot\left(x_2 - \frac{d}{2}\right)^2$$

It will be assumed that the shearing strength of a rivet is equal to the strength of a beam to resist cross-breaking.

(11).—*Single-riveted lap and single cover joints* (*Figs.* 15 *and* 16).

$$\frac{\pi.d^2}{4}.f_2 = (p-d).t.f_1 = d.t.f_4 \qquad (1)$$

$$2.x_1.t.f_2 = \frac{\pi.d^2}{4}.f_2, \qquad \therefore x_1 = \frac{\pi}{8}.\frac{d^2}{t} \qquad (2)$$

$$\frac{4}{3}.\frac{f.t}{d}.\left(x_2-\frac{d}{2}\right)^2 = \frac{\pi.d^2}{4}.f_2, \qquad \therefore x_2 = \frac{d}{2}.+\frac{1}{4}.\sqrt{3.\pi.\frac{d^3}{t}.\frac{f_2}{f}} \qquad (3)$$

(12).—*Single-riveted double cover joints*, (*Fig.* 17).

$$2.\frac{\pi.d^2}{4}.f_2 = (p-d).t.f_1 = d.t.f_4 \qquad (4)$$

$$2.x_1.t.f_2 = 2.\frac{\pi.d^2}{4}.f_2, \qquad \therefore x_1 = \frac{\pi}{4}.\frac{d^2}{4} \qquad (5)$$

$$\frac{4}{3}.\frac{f.t}{d}.\left(x_2-\frac{d}{2}\right)^2 = \frac{1}{2}.\frac{\pi.d^2}{4}.f_2, \quad \therefore x_2 = \frac{d}{2}+\frac{1}{4}.\sqrt{\frac{3}{2}.\pi\frac{d^3}{t}.\frac{f_2}{f}} \qquad (6)$$

Note.—The joints in § (11) are weakened by the bending action, possibly also by the concentration of the stress towards the inner faces of the plates, and experiment shows that they are not nearly so strong as the joints in §(12) of the same *theoretic* strength. The second cover in the latter must necessarily diminish the effect of the bending and preserve a more uniform distribution of stress. In all probability, the equality is restored by riveting a stiffener (*e.g.*, an angle-iron) to the plates in the lap or single cover joint, and in any case the difference will wholly disappear if the joint is double riveted.

Again, equations (1) and (4) shew that the bearing unit-stress is twice as great in the joints of § (12) as in the joints of § (11), so that rivets of a larger diar. may be employed in the latter than is possible in the former, for corresponding values of $\frac{d}{t}$.

(13).—*Chain riveted joints*, (Fig. 19).

$$f_1.(w-n.d).t = S = f_4.N.d.t \qquad (7)$$

$$S = N.\frac{\pi.d^2}{4}.f_2, \text{ when there is } \textit{one} \text{ cover only.} \qquad (8)$$

$$S = N.\frac{\pi.d^2}{2}.f_2, \text{ when there are } \textit{two} \text{ covers.} \qquad (9)$$

This class of joint is employed for the flanges of bridge girders, the plates being piled as in Figs. 21, 22, 23, and n being usually 3, 4 or 5.

In Fig. 22, the plates are grouped so as to *break joint*, and opinions differ as to whether this arrangement is superior to the *full butt* shewn

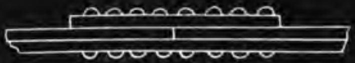


FIG. 21

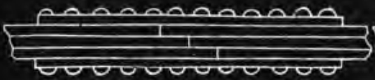


FIG. 22

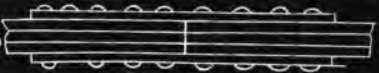


FIG. 23

in Fig. 23. The advantages of the latter are that the plates may be cut in uniform lengths, and the flanges built up with a degree of accuracy which cannot be otherwise attained, while the short and awkward pieces accompanying broken joints are dispensed with.

A good practical rule, and one saving much labour and expense, is to make the lengths of the plates, bars, etc., multiples of the pitch, and to design the covers, connections, etc., so as to interfere with the pitch as little as possible.

The distance between two consecutive joints of a group, (Fig. 22), is generally made equal to *twice* the pitch.

(14).—*Zig-zag and other riveted joints* (*Fig.* 20).—

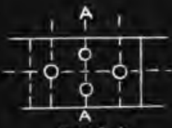


FIG. 24

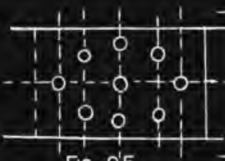


FIG. 25

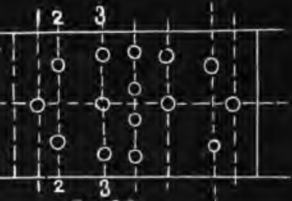


FIG. 26

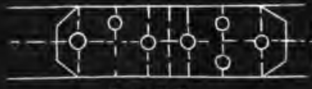


FIG. 27

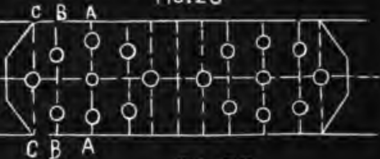


FIG. 28

A few experiments indicate that *chain* is a little stronger than *zig-zag* riveting. An excellent plan for lap and single cover joints is to arrange the rivets as shewn in Figs. 24 to 28. In reality, the strength of the plate at the joint is only weakened by *one* rivet hole, for the plate cannot tear at its weakest section, *i.e.*, along the central row of rivets (*aa*), until the rivets between it and the edge are shorn in two.

Let there be m rows of rivets, 11, 22, 33...... (Fig. 26).

The total number of rivets is evidently m^2.

Let f_1, q_2, q_3, q_1......, be the unit tensile stresses in the plate along the lines, 11, 22, 33,......,respectively.

$$\therefore S = (w-d).t.f_1 = \frac{\pi.d^2}{4}.m^2.f_2, \quad \text{for the line 11}$$

$$(w-2.d)t.q_2 = \frac{\pi.d^2}{4}.(m^2-1).f_2, \quad \text{“} \quad \text{“} \quad 22$$

$$(w-3d).t.q_3 = \frac{\pi.d^2}{4}.(m^2-3).f_2, \quad \text{“} \quad \text{“} \quad 33$$

$$(w-4.d).t.q_4 = \frac{\pi.d^2}{4}.(m^2-6).f_2, \quad \text{“} \quad \text{“} \quad 44$$

$$\cdots\cdots \qquad \cdots\cdots \qquad \cdots\cdots \qquad \cdots\cdots$$

$$\therefore S = (w-d).t.f_1 = (w-2.d).\frac{m^2}{m^2-1}t.q_2 = \cdots\cdots\cdots$$

Assume that $f_1 = q_2$, $\therefore w = (m^2+1).d$.

Hence, by substituting this value of w in the first of the above relations, $\frac{d}{t} = \frac{14}{11}.\frac{f_1}{f_2}$, since q_3, q_4...... are each less than f_1, so that the assumption is justifiable.

(15).—*Covers.*—In *tension* joints the strength of the covers must not be less than that of the plates to be united. Hence, a *single* cover should be at least as thick as a single plate, and if there are two covers, each should be at least half as thick.

When two covers are used in a tension pile it often happens that a joint occurs in the top or bottom plate, so that the greater portion of the stress in that plate may have to be borne by the nearest cover. It is, therefore, considered advisable to make its thickness $\frac{5}{8}$-ths. that of the plate.

The number of the joints should be reduced to a minimum, as the introduction of covers adds a large percentage to the dead weight of the pile.

Covers might be wholly dispensed with in *perfect jump* joints, and a great economy of material effected, if the difficulty of forming such joints and the increased cost did not render them impracticable. Hence, it may be said that covers are required for all *compression* joints, and that they must be as strong as the plates, for, unless the plates butt closely, the whole of the thrust will be transmitted through the covers. In some of the best examples of bridge construction, the tension and compression joints are identical.

(16).—*Rivet connexion between flanges and web.*

The web is usually riveted to the angle-irons which form part of the flanges.

Let M be the bending moment, and S the shearing force, at the point of which the abscissa is x, x being measured along the girder from any given origin.

Let h be the depth of the girder.

Let f_2 be the safe shearing stress per unit of area.

Two cases may be considered.

Case I.–Plate Web.—The shearing area of the rivets connecting the web with the angles must be equal to the theoretic horizontal section of the web, *i.e.*, to $\frac{S}{f_2.h}$, and the corresponding number of rivets per unit of length $=\frac{\frac{S}{f_2.h}}{\frac{\pi.d^2}{4}}=\frac{14}{11}.\frac{1}{n.d^2}.\frac{S}{f_2}$

Let F be the horizontal stress transmitted to the angles through the web, and tending to make them slide over the flange face.

$\therefore F.h$, the *increment* of the bending moment, $=\frac{dM}{dx}=S$, or $F=\frac{S}{n}$.

The requisite shearing area $=\frac{S}{f_2.h}$ and number of rivets is therefore the same as before.

Case II.–Open Web.—One of the braces which meet at each apex is a strut, and the other a tie.

The sectional area of the rivets connecting each brace with the angle-iron must be equal to the *net* section of that brace.

Let F be the horizontal component of the diagonal stresses.

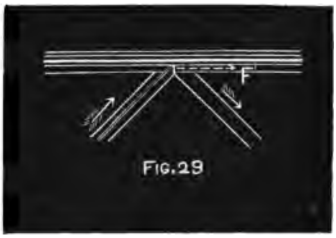


FIG. 29

$\therefore$ $F.h$ the increment of the bending moment, $=S$

$\therefore$ $F=\frac{S}{h}$, is the force which tends to make the angle slide over the flange face.

Hence, the number of rivets between two consecutive apices.

$$=\frac{14}{11}.\frac{1}{h.d^2}.\frac{S}{f_2}.$$

EXAMPLES.

(1).—A trellis girder rests on two supports without exerting any lateral pressure upon them. A vertical section of the girder meets two sets of n bars inclined in opposite directions, and it is assumed that in each set a mean stress may be substituted for the different stresses in the bars. If D, D', are the mean stresses and T, C, the horizontal forces in the flanges at the section, shew that $n.(D-D').\sin\theta + T - C$ must be *zero* at all points of the span, independently of any additional hypothesis, θ being the common inclination of the bars to the vertical.

Note.—If $T = C$, D must necessarily be equal to D'.

(2).—The tensile and compressive unit-stresses in the chords of a bridge-truss of span l and uniform depth d, are nowhere to exceed f_1 and f_2, respectively; shew that the greatest central deflection is approximately $(f_1+f_2)\cdot\frac{l^2}{8.E.d}$. Hence, if the span is eight times the depth, and if the grade line is to be truly horizontal when the bridge is loaded, shew that the length of the top chord should exceed that of the bottom chord by an amount equal to the camber.

(3).—A lattice girder 200-ft. long and 20-ft. deep, with two systems of right-angled triangles, carries a dead load of 800-lbs. per lineal ft.; determine the greatest stresses in the diagonals of the fourth bay from one end, when a live load of 1200-lbs. per lineal ft. passes over the girder.

(4).—A lattice girder 80-ft. long and 8-ft. deep carries a uniformly distributed load of 144,000-lbs.; find the flange inch-stresses at the centre, the sectional area of the top flange being 56½-sq. ins. gross, and of the bottom flange 45-sq. ins. net.

What should be the camber of the girder, and what extra length should be given to the top flange, so that the bottom flange of the loaded girder may be truly horizontal?

(5).—A lattice girder 80-ft. long, and 10-ft. deep, with *four* systems of right-angled triangles carries a dead load of 1000-lbs. per lineal ft.; determine the greatest stresses in the diagonals met by a vertical plane in the *seventh* bay from one end, when a live load of 2500-lbs. per lineal ft. passes over the girder. Design the flanges, which are to consist of plates riveted together.

The lattice bars are riveted to angle irons; find the number of $\frac{7}{8}$-in. rivets required to connect the angle-irons with the flanges in the first bay, 10,000-lbs. per sq. in. being the safe shearing strength of the rivets.

(6).—The bracing of a lattice girder consists of a single system of triangles in which one of the sides is a strut and the other a tie inclined to the horizontal at angles of α and β, respectively; in order to give the strut sufficient rigidity its section is made k-times that indicated by theory, the co-efficient k being $>$ unity; shew that the amount of material in the struts and ties is a minimum when $\frac{\tan \alpha}{\tan \beta} = k$.

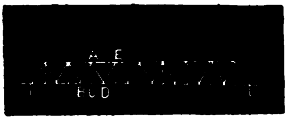
(6).—A bridge of n equal spans crosses a space of L-ft.; w_2 and w_3 are respectively the live load and weight of the platform, permanent way, etc., in tons per lineal ft.; shew that the weight w_1 of the main girders in tons per lineal ft. may be suppressed in the form

$$\frac{L.\,w_2.\,(p.\,k+r)\;+\;w_3.n}{n-L.\,(p.\,k+q)},$$

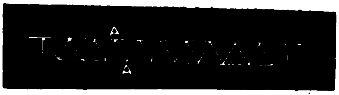
k being the ratio of the span to the depth, and p, q, r, numerical co-efficients. Hence, determine the limiting span of a girder.

If X is the cost of each pier, Y the cost per ton of the superstructure, determine the value of n which will make the *total* cost per lineal ft., *i.e.*, $\frac{(n-1).X}{L} + w_1.Y$, a minimum, and prove that this is approximately the case where the spans are so arranged that the cost of a span is equal to that of a pier.

(8).—A warren girder with its bracing formed of *nine* equilateral triangles, and with every joint loaded, is 90-ft. long; its dead weight is 500-lbs. per lineal ft.; prepare a table shewing the maximum stress in each bar and bay when a live-load of 1350-lbs. per lineal ft. crosses the girder, allowing for an engine-excess of 4,000-lbs. per panel, extending over two consecutive panels. The diagonals and verticals are riveted to angle-irons forming part of the flanges; how many $\frac{3}{4}$-in rivets are required for the connection of AB, AC, and AD, at A? Also, how many are required between A and E to resist the tendency of the angle-irons to slip longitudinally?

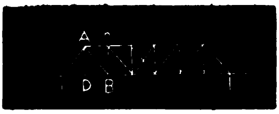


(9).—If a force of 5000-lbs. strike the bottom chord of the girder in the preceding question at 20-ft. from one end, and in a direction inclined at 30° to the horizontal, determine its effect upon the several members.



(10.)—The platform of a single-track bridge is supported upon the top chords of two warren girders; each girder is 100-ft. long, and its bracing is formed of *ten* equilateral triangles; the dead weight of the bridge is 900-lbs. per lineal ft.; the greatest *total* stress in A A when a train crosses the bridge is 41,394.8-lbs.; determine the weight of the live load per lineal ft. Prepare a table shewing the greatest stress in each bar and bay when a single load of 15,000-lbs. crosses the girder.

(11).—The compression and tension bars in certain two bays of a lattice girder are required to bear stresses of 90-cwts. and 130-cwts respectively; design the bars and specify the size and number of the rivets at the attachments, allowing 100-cwts. per sq. in. of *net* section in tension, 65-cwts. per sq. in. of *gross* section in compression, and a shearing stress on the rivets of 100-cwts. per sq. in.



(12).–The two trusses for a 16-ft. roadway are each 100-ft. in the clear, 17-ft. 3-in. deep, and of the type represented in the Fig.; under a live load of 1120-lbs. per lineal ft. the greatest total stress in *AB* is 35,400-lbs., determine the permanent load.

The diagonals and verticals are riveted to angle-irons forming part of the flanges; how many $\frac{3}{4}$-in. rivets are required for the connection of *AB* and *BC* at *B*? also, how many are required between *B* and *D* to resist the tendency of the angle-irons to slip longitudinally?

(13).—Design a double flanged plate girder, 8-ft. deep and 80-ft. long, to carry per lineal ft. a dead load of 500-lbs. and a live load of 1600-lbs., the safe tensile and compressive inch-stress being 10,000-lbs. and 8,000-lbs., respectively.

Determine the number of rivets required per lineal unit of length at the end and centre of the girder to connect the flange angle-irons with the web.

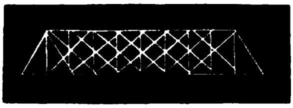
(14).—Find the points in each chord of the girder in the preceding question at which the chord stress is equal to the vertical shearing stress.

(15).—A Howe truss, 80-ft. long and 8-ft. deep, has a single set of diagonals at 45°; find the chord and diagonal stresses in the 3rd bay: (*a*)–When the truss carries a uniformly distributed load of 40-tons, (*b*)–When the 40-tons are concentrated at the middle of the span. (*Neglect the weight of the truss*).

(16).—Design a 16-panelled Howe truss for a clear span of 200-ft., and of such a depth that the chord stress in the 8th and 9th panels is one-half of the total distributed load (dead load per lineal ft.=800-lbs. live load per lineal ft.=1200-lbs.).

(17).—What should be the depth of the truss in the preceding question so that the chord and diagonal stresses in the second panel may be equal?

(18).—A Howe truss with *N*-panels and a span of *l*-ft. is designed for a dead load of *w*-lbs. and a live load of *w'*-lbs. at each of the panel points in the bottom chord; assuming that the sectional areas of the different members are directly proportional to the stresses to which they are subjected, determine the depth of truss which will secure the greatest economy of material.



(19).—The platform of a double-track railway bridge at Thamesville is supported from the bottom chords of two trusses braced and counter-braced as in the Fig. The length of each truss is 184-ft. 2-in., its depth is 34-ft., and it is designed to carry per lineal ft. a live load of 2250-lbs. and a dead load of 1100-lbs.; prepare a table shewing the max. stress in each member.

(20).—Design a cross-tie for the bridge in the presiding question, the live load for the floor-system being 8000-lbs. per lineal ft.

(21).—A 16-panelled Murphy-Whipple truss 200-ft. long, and 18-ft. 9-ins. deep, is designed to carry per lineal ft. a dead load of 750-lbs. and a live load of 1200-lbs., the panel excess weight due to the engine being 4000-lbs., and that due to the tender 3000-lbs.; prepare a table shewing the max. stress in each member, and specify the bars which require to be counter-braced.

(22).—Determine the greater weight that may be constructed at 50-ft. from one end of the truss in the preceding question.

(23).—Design a quadrangular truss (See Fig.) with square panels for a span of 91-ft., the chords and posts being of timber and the ties of wrought-iron.

(24).—Design a Fink truss for a 242-ft. span from the following data:—depth of truss = 30-ft.; length of panel = 15-ft. 1½-ins.; dead load per lineal ft. of truss = 1232-lbs.; live load per lineal ft. of truss = 1300-lbs.; the engine excess weight upon each of the end posts = 1704-lbs.

(25).—A_1, A_2, and a_1, a_2, are respectively the sectional areas and inclinations to the vertical of the two ties which meet at the foot of a post in a Bollman truss; if the sectional area of each tie is proportioned to the stress to which it is subjected, shew that

$$A_1. \cos^2 a. \sin a = A_2. \cos^2 a_2. \sin a_2.$$

(26).—Design an eight-panelled Bollman truss, 100-ft. long and 12½-ft. deep, to carry a uniformly distributed load of 200-tons, together with a single load of 10-tons concentrated at 25-ft. from one end.

(27).—In a Post truss for a 200-ft. span there are 16-panels, the posts incline towards the centre, and have a run of half a bay, the ties cross two panels, and are inclined at 45° to the vertical, the counter-ties cross one panel, and have the same inclination, the panel weight of engine = 17,600-lbs., of tender = 16,160-lbs., and of cars = 12,500-lbs.; the maximum stress in the *seventh* tie from one end = 56,000-lbs.; determine the panel weight of truss, and specify the bars which require to be counter-braced.

(28).—A weight is placed at the centre of a truss of which the web consists of a single system of struts and ties; shew how to determine the inclination of the web members to the vertical, so that the amount of material in them may be a minimum.

(29).—Design a bowstring girder, (*a*)—with vertical posts and diagonal bracing, (*b*)—with isosceles bracing, from the following data:—
The span is 80-ft., the depth at the centre is 10-ft., the dead load per ft. run is ½-ton, the live load per ft. run is 1-ton.

(30).—The bowstring girder represented by the Fig. is designed to support a load of 840-lbs. per ft. run; its span is 100-ft., its central depth 14-ft., its depth at each end 5-ft., and its upper chord is a circular arc; determine the stresses in all the members.

(31).—The span of a suspension bridge is 200-ft., the dip of the chains is 80-ft., and the weight of the roadway is 1-ton per ft. run; find the tensions at the middle and ends of each chain.

(32).—Assuming that a rope (or single wire) will safely bear a tension of 15-tons per sq. in., shew that it will safely bear its own weight over a span of about one mile, the dip being $\frac{1}{14}$-th of the span.

(33).—Shew that a steel wire rope of the best quality, with a dip of $\frac{1}{7}$-th of the span, will not break until the span exceeds 7 miles.

(34).—The platform of a suspension bridge of 150-ft. span is suspended by 90 vertical rods (45 on each side) from the cables, which have a dip of 15 ft.; the weight of the platform is 2240-lbs. per lineal ft.; find the stresses at the middle and ends of the cables, and in the suspenders, when a uniformly distributed load of 78,750-lbs. covers half the bridge.

(35).—Design a pair of auxiliary girders for the bridge in the preceding question, to counteract the effect of a live load of 1050-lbs. per lineal ft., and examine the effect of hinging the girders at the centre.

(36).—A bridge 442-ft. long consists of a central span of 180-ft. and two side spans each of 131-ft.; each side of the platform is suspended by vertical rods from two iron wire cables; each pair of cables passes over two masonry abutments and two piers, the former being 24-ft. and the latter 40-ft. above the surface of the ground; the lowest point of the cables in each span is 19-ft. above the ground surface; at the abutments the cables are connected with straight wrought-iron chains, by means of which they are attached to anchorages at a horizontal distance of 66-ft. from the foot of each abutment; the dead weight of the bridge is 3500-lbs per lineal ft., and the bridge is covered with a proof load of 4500-lbs. per lineal ft.; determine,

(*a*).—The stresses in the cables at the points of support and at the centre of each span.

(*b*).—The dimensions and weight of the cables, (1)—if of uniform section throughout, (2)—if each section is proportioned to the pull across it.

(*c*).—The alteration in the length of the cables and the corresponding depression of the platform at the centre of each span, due (1)—to a change of 60° F. from the mean temperature, (2)—to the proof load.

(*d*).—The crushing and bending efforts at the foot of a pier.

(*e*).—The bearing area at the top of the abutments, and the mass of masonry necessary to resist the tendency to overturning and to horizontal displacement. (*Weight of masonry per cubic ft.* = 128 *lbs., its safe compressive strength per sq. ft.* = 200-*lbs., its coefficient of friction* = .76).

(37).—Solve the preceding question when the cables for the several spans are independent, and have their ends securely attached to the points of support. The piers are wrought-iron oscillating columns, and in order to maintain the equilibrium of the structure under an unequally distributed loads the heads of the columns are connected with each other and with the abutments by iron wire stays; determine the proper dimensions of the stays, assuming then to be approximately straight.

(38).—The river span of a suspension bridge is 930-ft., and weighs 5976-tons, of which 1439-tons are borne by stays radiating from the summit of each pier, while the remaining weight is distributed between *four* 15-in. steel wire cables producing in each at the piers a tension of 2064-tons; find the dip of the cables.

The estimated maximum traffic upon the river span is 1311-tons, uniformly distributed; determine the increased stress in the cables.

To what extent might the traffic be safely increased, the limit of elasticity of a cable being 8116-tons, and its breaking stress 12,300-tons?

(39).—A semi-circular rib, pivoted at the crown and springings, is loaded uniformly per horizontal unit of length; determine the position and magnitude of the maximum bending moments, and shew that the horizontal thrust on the rib is *one-fourth* of the total load.

(40).—Draw the equilibrium polygon for an arch of 100-ft. span and 20 ft. rise, when loaded with weights of 3, 2, 4, and 2 tons respectively, at the end of the 3rd, 6th, 8th, and 9th division from the left support, of ten equal horizontal divisions. (*Neglect the weight of the rib*).

If the rib consist of a web and of two flanges $2\frac{1}{2}$ ft. from centre to centre, determine the maximum flange stress.

(41).—A parabolic rib of span l and rise k, having its ends A and B fixed, supports a given load at a horizontal distance x from the middle of the span; the equilibrium polygon consists of two straight lines CD, CE, meeting the verticals through A and B in D an E respectively; if $AD=y_1$, $BE=y_2$, and if y is the vertical distance of C from AB, shew that,

$$y_1=\frac{2}{15}\cdot\frac{l+10.x}{l+2.x}\cdot k,\quad y_2=\frac{2}{15}\cdot\frac{l-10.x}{l-2.x}\cdot k,\quad Y=\frac{6}{5}\cdot k.$$

(42).—A parabolic double-flanged rib $2\frac{1}{2}$-ft. deep, of 100-ft. span and 20-ft. rise, is fixed at both ends and loaded in the same manner as the rib in Question 40; draw the equilibrium polygon, and determine the flange stresses at the ends and at the division points.

(43).—A rib in the form of the segment of a circle, of span l and rise k, is hinged at its ends A,B, and supports a weight W at a horizontal distance x from the middle of the span: the equilibrium polygon consists of two straight lines CA, CB, the vertical distance of C from AB being Y; determine the value of Y, and shew that the horizontal thrust on the rib is $W.\frac{l^2-4.x^2}{4.l.Y}$.

(44).—If a rib in the form of the segment of a circle be substituted for the rib in Question (41), shew how to find y_o, y_1, Y, and apply to the case of semi-circular rib.

(45).—Draw the curve of equilibrium for a semi-circular rib of uniform section under a load uniformly distributed per horizontal unit of length, (*a*)—when hinged at the ends, (*b*)—when hinged at the ends and centre, (*c*)—when fixed at the ends.

(46).—Determine the horizontal thrust on a rib in the form of the segment of a circle due to change of t^o from the mean temperature.

(47).—In Chapter X, determine the changes to be made in the analysis, (*a*)—of § 13,14,15, when the equations deduced in Corollary 4, §(13), are employed instead of equations 10, 11 and 12.

(*b*).—Of § 13 and 14, when the abutments yield to the thrust so as to enlarge the span by an amount $\mu.H.$, μ being a co-efficient to be determined by experiment.

(*c*).—Of § 14 and 15, when the effect of a change of temperature is to be taken into account.

(48).—Illustrate § 13, 14, 15, by the example of a rib in which $k=\frac{l}{8}=5.z$, and $q=\frac{1}{6}$.

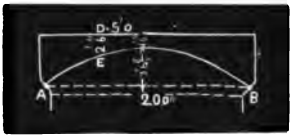
(49).—A wrought-iron parabolic rib, of 96-ft. span and 16-ft. rise, is hinged at the two abutments; it is of a double *T*-section, uniform throughout, and 24-ins. deep from centre to centre of the flanges; determine the compression at the centre, and also the position and amount of the maximum bending moment, (*a*).—when a load of 48-tons is concentrated at the centre, (*b*).—when a load of 96-tons is uniformly distributed per horizontal unit of length.

Determine the deflection of the rib in each case.

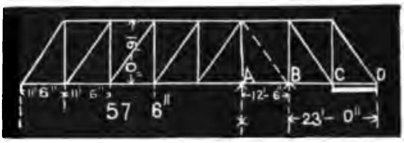
(50).—In the framed arch represented by the Fig., the span is 120-ft., the rise 12-ft., the depth of the truss at the crown 5-ft., the fixed load at each top joint 10-tons, and the moving load 10-tons; determine the maximum stress in each member with any distribution of load.

Shew that, approximately, the amount of metal required for the arch : the amount required for a bowstring lattice girder of the same span and 17-ft. deep at the centre: the amount required for a girder of the same span and 12-ft. deep :: 100 : 155 : 175.

(51).—Design an arched rib of 100-ft. span and 20-ft. rise, the fixed load being 40 tons uniformly distributed per horizontal unit of length, and the moving load 1-ton per horizontal ft.

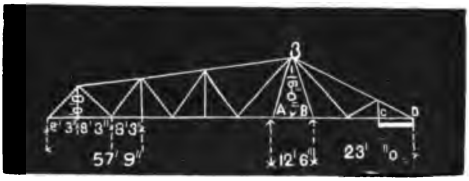


(52).—A cast iron arch, (see Fig.), whose cross-sections are rectangular, and uniformly 3-ins. wide, has a straight horizontal extrados, and is hinged at the centre and at the abutments. Calculate the normal intensity of stress at the top and bottom edges D, E, of the vertical section, distant 5-ft. from the centre of the span, due to a vertical load of 20-tons concentrated at a point distant 5-ft. 4-ins., horizontally from B; also find the maximum intensity of the shearing stress on the same section, and state the point at which it occurs. (AB= 21 ft. 4 ins.)



(53).—The sketch shews the truss of a counterbalanced swing-bridge, supposed to be wholly supported by the turntable at A and B; the dead weight is 650-lbs. per lineal ft. of the bridge, and the counterpoise consists of concrete hung in a box from the points C and D; required the weight of the counterpoise assuming, *first*, that it is all carried on B, and, *second*, that a portion of it is transmitted to A by the dotted bar, so that the reaction at A may be equal to that at B, and the whole weight of the bridge be equally distributed over the turntable.

Shew how to find the stresses in the different members of the truss.



(54).—The truss for a counterbalanced swing-bridge is of the form and dimensions shewn by the Fig., the counterpoise consisting of concrete hung in a box from the points C and D; the dead weight is 650-lbs. per lineal ft. of the bridge; determine the stresses in the different members, (*a*).—when the bridge is open and is wholly supported by the turntable at A and B, (*b*)—when the bridge is closed, and is subjected to a passing load of 3000-lbs. per lineal ft.

Design the joint at E so that, however unequally the arms may be loaded, the whole of the load may be transmitted to the main posts EA and EB and evenly distributed over the turntable.

(55).—A girder bridge of 100-ft. span deflects 3-ins. below the horizontal line under a certain load W, at rest; find the increased pressure due to centrifugal force if W crosses the bridge at 60-miles an hour.

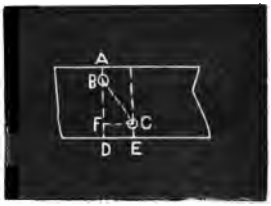
(56).—A stiffening girder for a suspension bridge has its ends fixed so as to be incapable of vertical motion.

If a load is concentrated at the centre of the girder, shew that the ratio of the vertical distance between the highest and lowest points of the central axis to the vertical distance between the end and lowest point is $\frac{9}{4}$.

If a uniformly distributed load covers *two-thirds* of the girder from one end, shew that the ratio of the vertical distance between the ends and the lowest point of the neutral axis to the vertical distance between the ends and the highest point is 4.

Determine the depression of the lowest point below the highest when *one-third* of the girder from one end is uniformly loaded.

(57).—The plate in Fig. is $7\frac{1}{2}$-ins. wide and $\frac{1}{2}$-in. thick; B and C are $\frac{7}{8}$-in. rivet holes; AB $= 1\frac{3}{4}$-ins. CF $=$ 3-ins.; find CE so that the efficiency on the line ABCE may be equal to that on the live ABD.



(58).—Determine the strength of a joint of the type represented by Fig. 28, Chap. VI., from the following data:—width of plate $= 4\frac{1}{16}$-ins., thickness of plates $=$ thickness of cover $= \frac{1}{2}$-in., diar. of rivets $= \frac{13}{16}$-ins., pitch of rivets on line AA $= 2\frac{7}{16}$-ins., on line BB $= 3\frac{1}{4}$-ins., distance between AA and BB $= 1\frac{1}{2}$-ins., between AA and CC $= 3\frac{1}{4}$-ins.

(59).—A $\frac{1}{2}$-in. flat diagonal bar at an angle of 45° is riveted to a $\frac{1}{2}$-in. vertical flange plate; if the admissible stress per sq. in. on the shearing area of rivet be 5-tons, and $7\frac{1}{2}$ tons on the bearing area, shew that the strength of the joint may be always made greater than that of the bar with two rows of rivets.

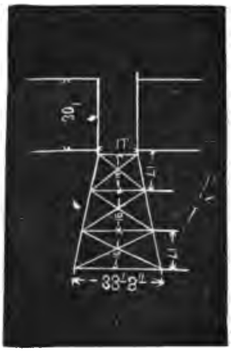
(60).—A riveted joint may fail by crushing, the bearing area being insufficient; assuming that the rivet and rivet hole retain their cylindrical forms when the joint is strained, and that the pressure at any point of the bearing is normal to the surfaces in contact and equal in intensity to $f.\cos a.$, a being the angle between the normal and the direction of the resultant stress, shew that $\frac{f}{F}=\frac{11}{14}$, F and f being respectively the maximum and mean intensities of the crushing pressure in the direction of the resultant stress.

(61).—A chord-pin joint is of the type represented by Fig. 7, Chap. VI.; the eight bars are each 5-ins. deep by 1-in. wide; the two innermost bars are 1-in. apart, and the spaces between the remaining bars are closed up; find the diar. of the pin, assuming that the ultimate unit stress in a bar is $\frac{16}{27}$-ths. of the actual stress in the pin at its circumference.

(62).–The curvature of a girder at a vertical section of depth h on one side of a chord pin joint is increased a-times by the deformation of the web; r is the radius of gyration of the section of the chord; if f be the coefficient of sliding friction, shew that the diameter of the pin which will just prevent rotation is approximately $\frac{4.a.r^2}{f.h}$

State the practical effect of this result.

(63).–Two $1\frac{1}{8}$ ins. lateral rods are coupled on a pin; the outer is subjected to a stress of $2\frac{1}{2}$-tons and the lever arm of its longitudinal component is $1\frac{1}{2}$-ins., the inner is subjected to a stress of $9\frac{1}{2}$-tons, and the lever arm of its longitudinal component is 1-in.; the stresses in the web members produce a couple having its axis inclined at 45° to the vertical, and a moment of 20-inch tons; determine the proper diameter of the pin.



(64)–*Data.*—A deck bridge consists of two independent spans of 200-ft., supported at one end by an iron braced pier 50-ft. high. The trusses are 30-ft. centre to centre of chords; the distance between the centre lines of the trusses is 17-ft.; the planes of the pier faces batter 2-ins. in 1-ft.; a horizontal section of the pier is square. The wind pressure is 40-lbs. per sq. ft.; 10-sq. ft. per lineal ft. of bridge and 10-sq. ft. per lineal ft. of train, are subject to wind pressure; the dead load per lineal ft. is

1600-lbs. and the live load 2800-lbs.; the assumed weight of the pier is 1800-lbs. per ft. in height; the cylinder of the locomotive is 16-ins. in diam., the pressure on the piston is 130-lbs. per sq in.:—

(*a*).—Find the stresses in the wind bracing.

(*b*).—If a train of empty cars is on the bridge and are on the point of overturning, ascertain whether the inclined posts will be subjected to tensile stresses. The cars weigh 900-lbs. per lineal ft., and the centre of pressure is 7-ft. above the rails.

(*c*).—Ascertain whether the wind pressure of 40-lbs. per sq. ft. upon the empty bridge will produce tension anywhere in the inclined posts, the centre of pressure being 3-ft. above the middle of the truss.

(*d*).—A loaded train rests upon one span, the engine being directly over the inclined posts; find the stresses in the traction bracing when the whole pressure of 130-lbs. per sq. in. is suddenly applied upon the piston.

(*e*).—A loaded train, 200-ft. in length, is running at a speed of 30 miles an hour. Sufficient power is applied to the car-brake to bring the train to rest in 300-ft.; find the stresses in the traction bracing when the engine is over a pair of inclined posts.

(*f*),—Find the max. stresses in inclined posts in each of the three sections, by the method of moments, neglecting the effect of traction.

(*g*).—Will the shoe at the foot of a post slide under any distribution of the load? (Coeff. of friction$=f$)

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